ROCZNIKI INŻYNIERII BUDOWLANEJ – ZESZYT 8/2008
Komisja Inżynierii Budowlanej Oddział Polskiej Akademii Nauk w Katowicach

ACCEPTABLE FATIGUE CRACK SIZE - THEORY

Prof. Ing. Vladimír TOMICA, CSc., Ing. Martin KREJSA, Ph.D.
VŠB-TU Ostrava

Ing. Jozef GOCÁL, Ph.D.
Faculty of Civil Engineering, ŽU Žilina

1. Introduction

Attention to the fatigue cracks in steel structures and bridges has been paid for a long time. In spite of efforts to eliminate the creation and propagation of fatigue cracks throughout the designed service life, cracks are still revealed during inspections. According to one of standardized methods in EC standards, propagation of cracks is acceptable to a certain extent. The standard defines certain general conditions that need to be fulfilled. No procedures, however, have been defined for the problem solution of the criteria.

Three sizes are important for the characteristics of the propagation of fatigue cracks. The first size is the initiation size, the second is the measurable length ant the third important size is referred to as a critical size. The critical size is the final size before a brittle fracture results in a failure.

Fatigue crack damage depends on a number of stress range cycles. This is a time factor in the course of reliability for the entire designed service life. The failure rate increases in the course of time and the reliability decreased. If possible propagation of the fatigue crack is included into the failure rate, it is necessary to investigate into the fatigue crack and define the maximum acceptable degradation. The acceptable size is a limit for the required reliability.

The topic is discussed in two levels: the probabilistic solution to the propagation of the fatigue crack and uncertainties in determination of values used in the calculation.

2. Propagation of the fatigue crack

The fatigue crack, that degraded a certain area of the structure components, is described with one dimension only a when investigating into the propagation. In order to describe the propagation of the crack, the linear elastic fracture mechanics is typically used defined by Paris-Erdogan law

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C.(\Delta K)^m , \tag{1}$$

where C, m are material constants, a is the crack size and N is the number of loading cycles.

The primary assumption is that the design should take into account the actions of the extreme loading and the fatigue resistance should be assessed then. The reliability reserve in the technical probability method is:

$$g_{(R,S)} = G = R - S, \qquad (2)$$

where R is the random resistance of the element and S represents random variable effects of the extreme load. If such element is subject to the service load, following cases can occur (Fig. 1):

a) safe life – the fatigue effects do not degrade the element by the fatigue crack,
b) damage tolerance – the fatigue effects degrade the element and decrease the load-bearing capacity of the element,
c) damage tolerance – fatigue effects indicated stress changes.

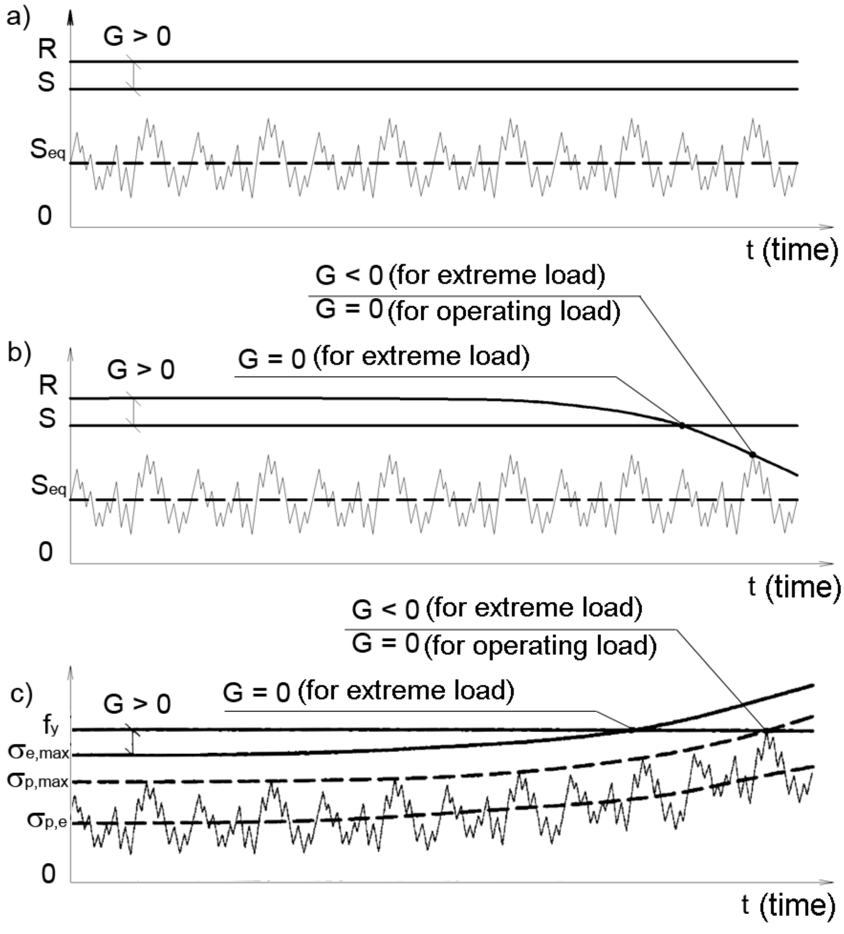

Fig. 1: Reliability reserve related to designed and operating load effects

When using (1), the condition for the acceptable crack size a_{ac} is:

$$N = \frac{1}{C}\int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\Delta K^m} > N_{tot}, \tag{3}$$

where N is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac}, and N_{tot} is the number of cycles throughout the service life.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. The range of the stress intensity factor ΔK to the constant stress range $\Delta\sigma$ is:

$$\Delta K = \Delta\sigma.\sqrt{\pi.a}.F_{(a)}. \tag{4}$$

The calibration function $F_{(a)}$ represents the course of propagation of the crack. Having modified (1) and using (4), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi.a}.F_{(a)}\right)^m} = \int_{N_1}^{N_2} C.\Delta\sigma^m.\mathrm{d}N \,\cdot \tag{5}$$

Left side of the equation (5) can be regarded as the resistance of the structure R, right side defined the cumulated effect of loads S.

It is possible to define a reliability function. The analysis of the reliability function gives a failure probability P_f:

$$G_{fail(Z)} = R_{(a_2)} - S, \tag{6}$$

where Z is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack. Probability of failure equals to:

$$P_f = P\left(G_{fail(Z)} < 0\right) = P\left(R_{(a_2)} < S\right). \tag{7}$$

3. Probabilistic calculation method

Let us define following random phenomena and their probabilities that are related to the growth of the fatigue crack and may occur in any time, t, during the service life of the structure:

- the probability that the failure occurs within the t-time, this means the probability that the fatigue crack size $a(t)$ reaches the acceptable size a_{ac}

$$P(F(t)) = P(a(t) \geq a_{ac}), \tag{8}$$

- the probability that the failure detected within the t-time has the crack size $a(t)$ that is less than the acceptable size a_{ac}

$$P(D(t)) = P(a_d \leq a(t) < a_{ac}), \tag{9}$$

- the probability that the failure is not detected within the t-time, this means the probability that the fatigue crack size $a(t)$ is below the measurable crack size a_d

$$P(U(t)) = P(a(t) < a_d). \tag{10}$$

Those three phenomena cover the complete spectrum of phenomena that might occur in the t-time. This means:

$$P(F(t)) + P(D(t)) + P(U(t)) = 1. \tag{11}$$

The probable course of the growth of the fatigue crack is shown in Fig. 2.

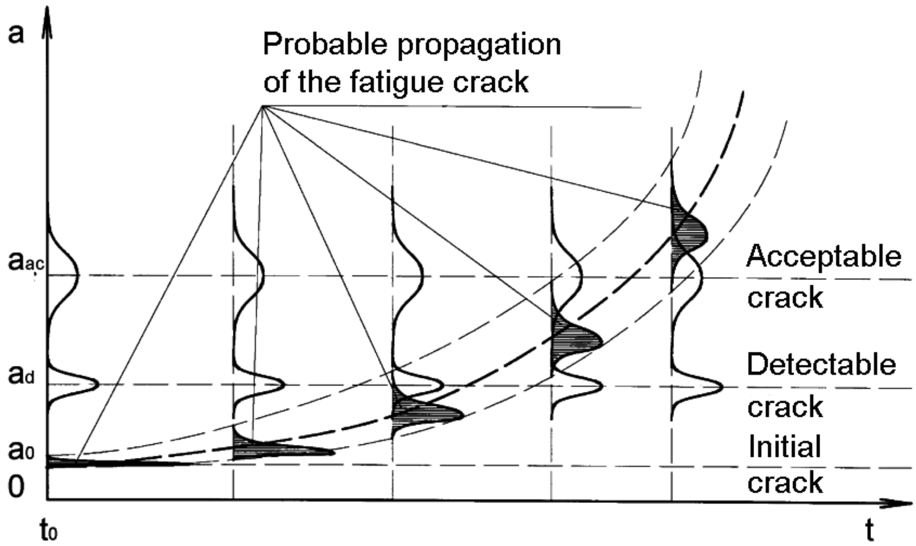

Fig. 2: Probabilistic propagation of the fatigue crack in the course of time

The probabilities in the equations (8) – (10) can be calculated in any t-time using the simulation methods based on Monte Carlo or the Direct Determined Fully Probabilistic Method [4] ("DDFPM"), for instance [1] and [2]). The calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the failure probability $P(F(t))$ reaches the designed failure probability P_{fd}, an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection provides information about real conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations. The inspection in the t-time may be one of the three events presented in equations (8) – (10).

Using the inspection results for t_I, it is possible to define the probability of the mentioned phenomena in another times: $T > t_I$. In order to determine the time for the next inspection, it is necessary to define the conditional probability:

$$P\left(F(T)/U(t_I)\right) = \frac{P\left(F(T)\right) - P\left(F(t_I)\right) - P\left(D(t_I)\right) \cdot P\left(F(T)/D(t_I)\right)}{P\left(U(t_I)\right)} \quad . \tag{12}$$

The probabilities in the equation (12) can be calculated in any $T > t_I$ time using the simulation methods based on Monte Carlo or DDFPM [4]. When the failure probability $P(F(t) / U(t_I))$ reaches the designed failure probability P_{fd}, an inspection should be carried out in order to reveal fatigue cracks, if any, in the construction component. The inspection may result in one of the mentioned phenomena with corresponding probabilities. The entire calculation can be repeated in order to ensure well-timed inspections in the future.

3. Used probabilistic methods and their comparison

Two probabilistic methods have been used in the calculation of (8) and (12) probabilities:

- Monte Carlo method
- Direct Determined Fully Probabilistic Method (DDFPM)

The first method has been presented several times [7]. For purposes of this study it has been modified and adapted. The second method has been chosen as a brand new method for this purpose.

The Direct Determined Fully Probabilistic Method ("DDFPM") [1] and [2] was originally developed as a Monte Carlo alternative to SBRA [8]. DDFPM is an alternative to Monte Carlo and can be used similarly.

The main benefit of DDPFM in the probabilistic calculations is that the results achieved are more accurate. DDFPM always provides an unambiguous and comparable result (this differs from the results achieved on the basis of the Monte Carlo method). The only error in this result is given by a numerical error and error resulting from discretization of input quantities. If Monte Carlo is used for the same calculation, the results will be always slightly different even if a relatively high number of simulations is used. The reason is the generation of random numbers, or to be more specific – pseudo-random numbers. This generation is always limited and slightly different for each series of simulations. If the directly probabilistic calculation is used and same intervals are chosen, the result is always the same.

4. Conclusion

This article provides theoretical backgrounds for propagation of fatigue cracks. A particular attention is paid to the maximum acceptable crack size. The final fatigue crack size may contribute to a division made between the critical crack size and acceptable crack size. The acceptable crack size comprises safety margins for the critical crack size that may occur in consequence of a brittle fracture and, more often in steel structures, in consequence of a ductile fracture.

Acknowledgements

This outcome has been achieved with the financial support of the Ministry of Education, Youth and Sports of the Czech Republic, project No. 1M0579, within activities of the CIDEAS research centre.

Literature

[1] JANAS, P., KREJSA, M., KREJSA, V. Structural Reliability Assessment Using Direct Determined Fully Probabilistic Calculation. In: *proceedings of 3rd International ASRANet Colloquium 2006* (abstract p.8, full paper on CD). 10 – 12th July 2006, Glasgow, UK. ISBN 0-9553550-0-1 / 978-0-9553550-0-4 (In English).

[2] JANAS, P., KREJSA, M., KREJSA, V. Posuzování spolehlivosti konstrukcí v programovém systému ProbCalc. In: *Polsko-Czeskie Sympozjum Naukowo-Techniczne*. Opole, Polsko, 2006. pp. 87-92. ISBN 83-923680-0-2.

[3] TALLIN, A. G., CESARE, M. Inspection Based Reliability Updating for Fatigue of Steel Bridges. In: *Proceedings of Bridge Management Conference*, London 1990.

[4] TOMICA, V., KREJSA, M. Optimal Safety Level of Acceptable Fatigue Crack. In: *5th International Probabilistic Workshop*. Ghent, Belgium, 28-29 November 2007. Edited by Luc Taerwe & Dirk Proske. ISBN 978-3-00-022030-2.

[5] TOMICA, V., KREJSA, M. Možnost pravděpodobnostního výpočtu únavové trhliny metodou PDPV. In: *Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava*. Řada stavební, číslo 1, rok 2007, ročník VII. pp.211-220. ISSN 1213-1962 ; ISBN 978-80-248-1661-6.

[6] TOMICA, V. Effect of Local Plasticity on Fatigue Resistance. In: *Building Research Journal*, SAP Ltd., Vol. 44, Nr. 4, Bratislava 1996, pp.253-263.

[7] KRÁLIK, J., KRÁLIK, J. Deterministický a pravdepodobnostný prístup k posudzovaniu spoľahlivosti výškových stavieb. In: *Stavebné a environmentálne inžinierstvo*, roč. 3, 2007, č.1, s. 12-27. ISSN 1336-5835.

[8] MAREK, P., GUŠTAR, M., ANAGNOS, T. Simulation – Based Reliability Assessment for Structural Engineers. CRC Press., Inc., 1995, U.S.A. ISBN 0-8493-8286-6.

ACCEPTABLE FATIGUE CRACK SIZE - THEORY

Summary

This article provides an introduction to the characteristics of the acceptable fatigue crack size in steel structures and bridges. This crack size plays a key role in degradation of an element designed for an extreme loading combination but in fact is exposed to variable service loads.