ROCZNIKI INŻYNIERII BUDOWLANEJ – ZESZYT 11/2011
Komisja Inżynierii Budowlanej Oddział Polskiej Akademii Nauk w Katowicach

NUMERICAL SIMULATION OF VEHICLE MOTION ALONG TWO SPAN BRIDGE STRUCTURE

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1. Introduction

Solution of the problems of vehicle bridge interaction belongs to the oldest solved problems of structural dynamics. The works of the civil engineer R. Willis [1] and mathematician G. G. Stoks [2] in which they tried to clarify the breakdown of Chester Rail Bridge in England in 1847 are not only considered the first attempts to solve the problems of vehicle runway interaction but also the first works in the field of structural dynamics. Slovak and Czech Republic are world-known by the high level of bridge engineering and theoretical approach to the solution of dynamical problems of bridges. Basic knowledge dedicated to the dynamic investigation of railway and highway bridges was published in monographs [3], [4]. Numerical methods offer an effective tool for the solution of this problem. Contemporary state of computing technique enables to solve all the problems in real time. The results obtained from numerical analyses are used in the process of the design of optimal parameters of bridges with respect to its lifetime and reliability.

2. Computing model of a vehicle and a bridge

For the purpose of this contribution the half plane computing model of the lorry TATRA 815 was adopted, Fig. 1. Equations of motion for 5 unknown functions $r_1(t) \div r_5(t)$ describing the vehicle vibration are derived in the shape of differential equations (1).

$$
\begin{gathered}
\ddot{r}_1(t) = -\{k_1 \cdot d_1(t) + b_1 \cdot \dot{d}_1(t) + k_2 \cdot d_2(t) + b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t)/\dot{d}_c\} / m_1 , \\
\ddot{r}_2(t) = -\{-a \cdot k_1 \cdot d_1(t) - a \cdot b_1 \cdot \dot{d}_1(t) + b \cdot k_2 \cdot d_2(t) + b \cdot b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t)/\dot{d}_c\} / I_{y1} , \\
\ddot{r}_3(t) = -\{-k_1 \cdot d_1(t) - b_1 \cdot \dot{d}_1(t) + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t)\} / m_2 , \qquad (1) \\
\ddot{r}_4(t) = -\{-k_2 \cdot d_2(t) - b_2 \cdot \dot{d}_2(t) - f_2 \cdot \dot{d}_2(t)/\dot{d}_c + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t) + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t)\} / m_3 , \\
\ddot{r}_5(t) = -\{-c \cdot k_4 \cdot d_4(t) - c \cdot b_4 \cdot \dot{d}_4(t) + c \cdot k_5 \cdot d_5(t) + c \cdot b_5 \cdot \dot{d}_5(t)\} / I_{y3} .
\end{gathered}
$$

The contact forces $F_{int,\mathrm{i},\ (\mathrm{i}\,=\,6,7,8)}$ belong to individual contact points are expressed as

$$F_{int,6}(t) = -G_6 + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t),$$
$$F_{int,7}(t) = -G_7 + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t), \tag{2}$$
$$F_{int,8}(t) = -G_8 + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t).$$

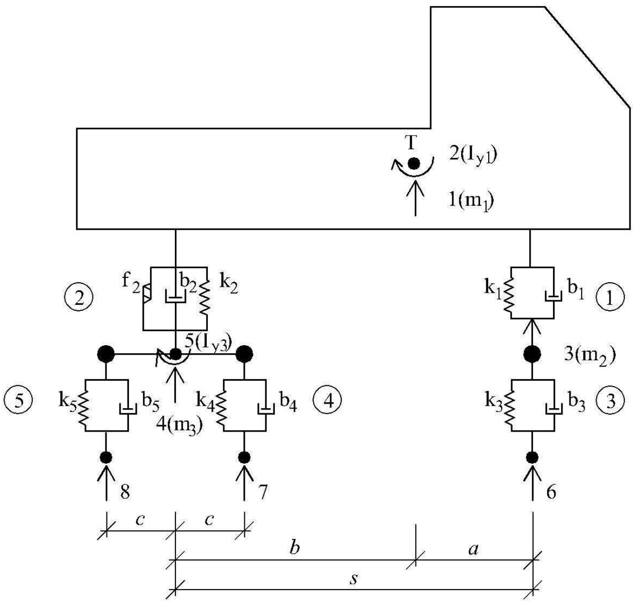

Fig. 1 Half plane model of the vehicle TATRA 815

For the bridge the beam computing model with two degrees of freedom was adopted. The assumption about the shape of deflection curve $v(x,t)$ in time moment t and the shape of load distribution function $\phi(x)$ was assumed in the shape of sine function, Fig. 2. Than

$$v(x,t) = \phi(x) \cdot v(t) + h(x) \text{ eventually } v_x(t) = \phi_x(t) \cdot v(t) + h(t), \tag{3}$$

where

$$\phi(x) = \sin(\pi \cdot x / l) \text{ eventually } \phi_x(t) = \sin(\pi \cdot e \cdot t / l) = \sin(\omega \cdot t), \; \omega = \pi \cdot e / l, \tag{4}$$

$$F(t) = F_{int}(t) \cdot \phi_x(t). \tag{5}$$

Equations of motion can be written as

$$[m]_D \cdot \{\ddot{v}(t)\} + 2 \cdot \omega_b \cdot [m]_D \cdot \{\dot{v}(t)\} + [k] \cdot \{v(t)\} = \{F(t)\}. \tag{6}$$

In the above expressions t is time coordinate, x length coordinate, $d(t)$ deformation of connecting members, G_i gravity force, $h(x)$, $h(t)$ road roughness, e speed of vehicle motion in [m/s], ω angular frequency, ω_b damping angular frequency. Derivatives of functions with respect to time are denoted by dot over symbol of dependently variable.

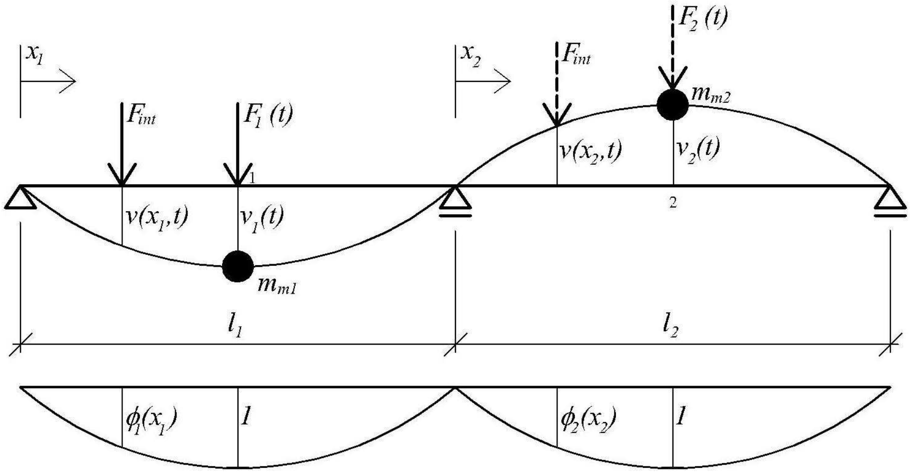

Fig. 2 Two span bridge computing model

3. Numerical simulation

For the numerical simulation of vehicle motion along the bridge structure the computer program in the program language MATLAB was created. The following input data for vehicle TATRA 815 and for the bridge were used during computation.
Vehicle input data and initial conditions:

k_1 = 287433 N/m; k_2 = 1522512 N/m; k_3 = 2550600 N/m; $k_4 = k_5$ = 5022720 N/m;
b_1 = 19228 kg/s; b_2 = 260197 kg/s; b_3 = 2746 kg/s; $b_4 = b_5$ = 5494 kg/s;
m_1 = 22950 kg; m_2 = 910 kg; m_3 = 2140 kg; I_{y1} = 62298 kg.m^2; I_{y3} = 932 kg.m^2;
a = 3,135 m; b = 1,075 m; s = 4,210 m; c = 0,660 m;

$r_1(0) = -0{,}02$ m; $\dot{r}_1(0) = 0{,}0$ m/s; $r_2(0) = 0{,}00$ rad; $\dot{r}_2(0) = 0{,}0$ rad/s;
$r_3(0) = -0{,}002$ m; $\dot{r}_3(0) = 0{,}0$ m/s; $r_4(0) = -0{,}003$ m; $\dot{r}_4(0) = 0{,}0$ m/s;
$r_5(0) = 0{,}00$ rad; $\dot{r}_5(0) = 0{,}0$ rad/s;

Bridge input data and initial conditions:

μ = 19680,0 kg/m; I = 1,60622 m^4; E = 3,85e10 N/m^2; ω_b = 0,1 rad/s;
m_{m1} = 285360,0 kg; l_1 = 29,0 m; m_{m2} = 285360,0 kg; l_2 = 29,0 m;

$v_1(0) = 0{,}0$ m; $\dot{v}_1(0) = 0{,}0$ m/s; $v_2(0) = 0{,}0$ m; $\dot{v}_2(0) = 0{,}0$ m/s;

The results displaying the time courses of vehicle and bridge vertical vibration while the lorry TATRA 815 passing the bridge at the speed 40 km/h are presented in the Fig. 3, 4, 5. The smooth road surface was assumed during computations.

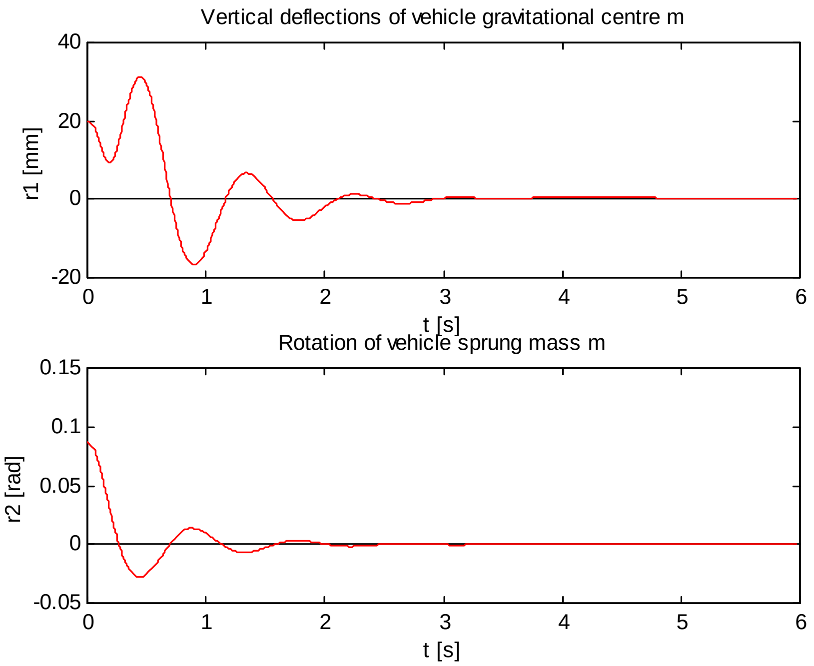

Fig. 3 Vibration of vehicle sprung mass

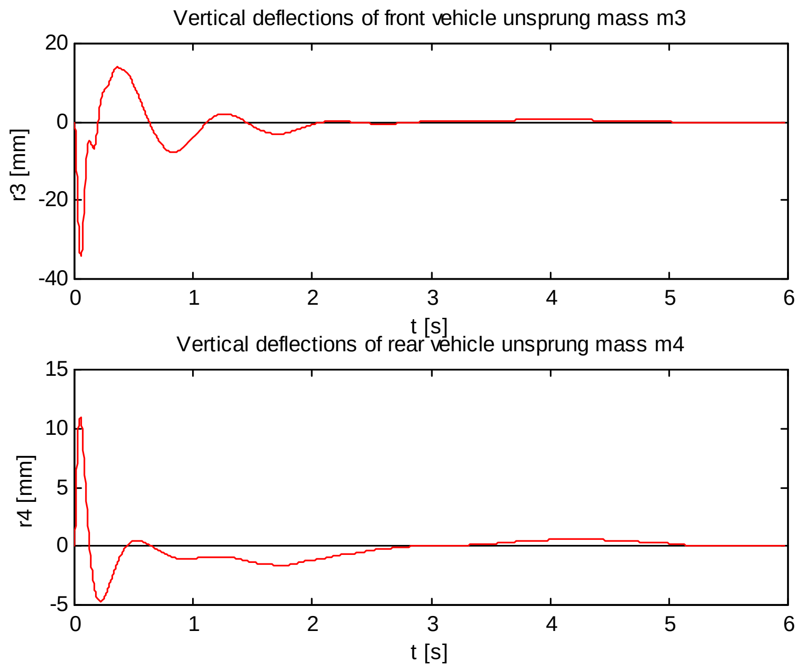

Fig. 4 Vibration of vehicle un-sprung masses

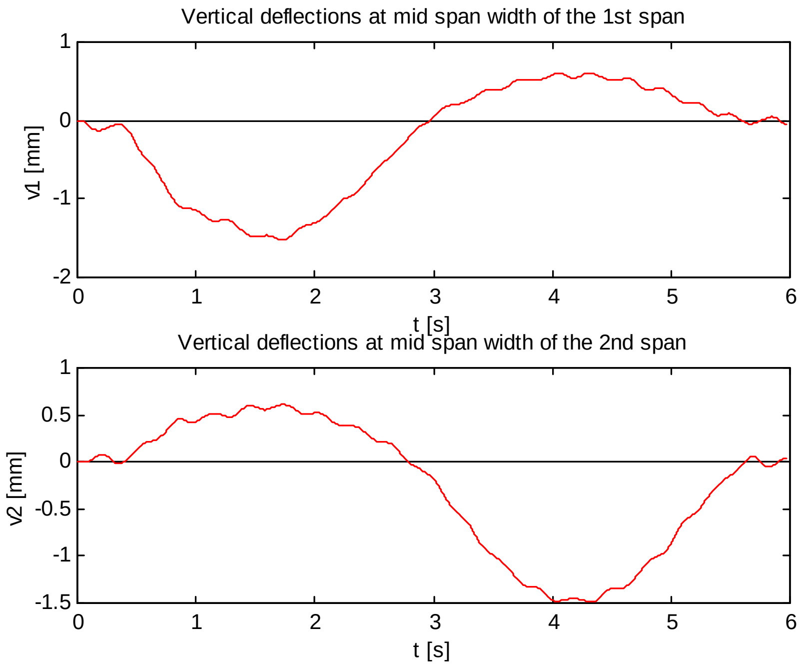

Fig. 5 Vibration of mid bridge spans

4. Conclusions

Numerical modeling of the problems of vehicle and bridge interaction is an effective tool for the solution of real tasks of engineering practice. Quality of obtained results is dependent on the quality of input data. The present state of computing technique enables the numerical processing of solved problems in real time. From the practical point of view the influence of various vehicle and bridge parameters is interested. The results obtained from numerical analyses are used in the process of design of optimal bridge parameters with respect to lifetime and reliability of the bridge structure.

References

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Acknowledgement

This contribution is the result of the project implementation: "Centre of Excellence in Transport Engineering" (ITMS: 26220120027) supported by the Research & Development Operational Programme funded by the ERDF.

The Authors are grateful for support from the Grant Agency VEGA of the Slovak Republic.

NUMERICKÁ SIMULÁCIA POHYBU VOZIDLA PO MOSTE S DVOMA POLIAMI

Resumé

Predkladaný príspevok je venovaný metódam numerického modelovania pohybu vozidiel po mostných konštrukciách. Uvažuje rovinný výpočtový model vozidla TATRA 815 a diskrétny výpočtový model mosta s dvoma poliami. Pri tvorbe diskrétneho výpočtového modelu mosta bol prijatý predpoklad o tvare ohybovej čiary a tvare funkcie popisujúcej roznos zaťaženia na jednotlivé hmotné body výpočtového modelu. Obidve funkcie sú zavedené do výpočtu ako sínusové funkcie. Riešenie pohybových rovníc je robené numericky v prostredí programovacieho jazyka vyššej úrovne MATLAB. Získané výsledky sú zobrazené v grafickej forme.