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OPTIMIZATION OF SANDWICH PLATES WITH TRUSS CORES

Ružica R. NIKOLIĆ,
Faculty of Engineering, University of Kragujevac, Serbia
Faculty of Civil Engineering, University of Žilina, Slovakia

Jelena M. DJOKOVIĆ,
Technical Faculty of Bor, University of Belgrade, Serbia

Jan BUJNAK
Faculty of Civil Engineering, University of Žilina, Slovakia
Faculty of Civil Engineering, the Opole University of Technology

1. Introduction

Sandwich plates with truss cores are widely applied due to the fact that they have the open structure, high stiffness and load carrying capacity. The new casting technologies enable manufacturing of the truss core elements within the wide range of dimensions, from several millimeters to hundreds time more. By electro-deposition forming of the truss structures at even smaller scales is enabled, with core elements whose diameters can be only fifty microns, Brittain et al. [1]. The well manufactured structure with the truss cores can be very efficient, from the standpoint of the mass reduction, i.e., the small weight, what is shown in the present work. These structures have a wide range of applications. For instance, they can be used simultaneously as the load carrying element in the construction and as an element for the heat transfer. The cavities between the plates can also be used as a container for liquid or as a pressure vessel for pressurized gas. The plates with the honeycomb cores, or the conventional truss structures, do not possess any of the aforementioned possibilities.

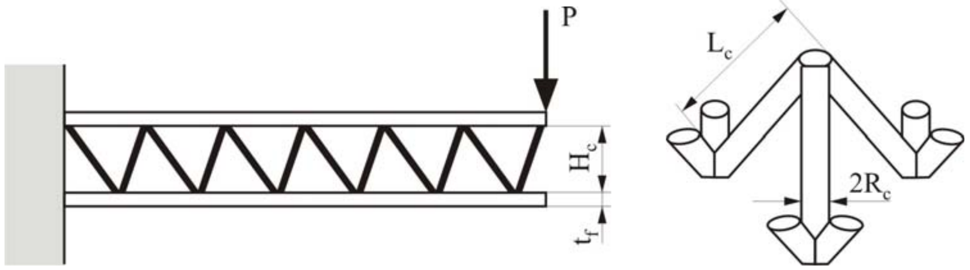
This paper presents an attempt to optimize the sandwich plate with truss cores, which is loaded by the bending moment and the transverse shearing force. The optimal configuration that is obtained is compared to the optimal plate with the honeycomb core.

2. Problem formulation

The sandwich plate with truss core that is considered in this paper is presented in Figure 1. The term "truss core" refers to a core constructed from beam elements, but not folded plates. The three types of the sandwich plates with honeycomb core are shown in Figure 2. The plate considered here has the tetragonal truss core, with the three-fold (120 °) planar symmetry, whose each member has the length L_c and the circular cross-section with radius

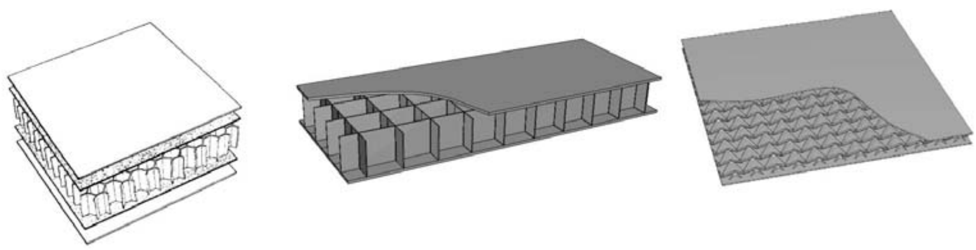
R_c. The core thickness is H_c. The 120 ° symmetry of plates ensures that their bending and in-plane stretching stiffnesses are isotropic, Allen [2]. The isotropic plate's thickness is t_f, while the angle between the core members and the faces (skins) is $\theta = \arcsin\left(H_c / L_c\right)$.

Here is assumed that both the plates and the truss core are made of the same material, which has the elasticity modulus E, Poisson's ratio ν, yield strength σ_y and material density ρ. The significantly more efficient structure can be obtained if for the plates and the truss core are adopted different materials.



(a) (b)

Fig. 1 Sandwich plate with the truss core: (a) Geometry; (b) Loading



(a) (b) (c)

Fig. 2 Sandwich plates with honeycomb core: (a) Honeycomb-like core; (b) rectangular core; (c) pyramidal core

The mass per unit area of the plate with truss core is:

$$W = 2\rho\left[t_f + \frac{\pi}{\sqrt{3}} \cdot \frac{L_c R_c^2}{R_c^2 - H_c^2}\right]. \tag{1}$$

In the general case, infinitely wide plate, shown in Figure 1, is exposed to maximum moment per unit length M and maximum transverse shear force per unit volume V. Bending occurs only about the direction which is parallel to the loading line. Each half of the plate carries a uniform transverse shear load per unit length P = V and a maximum moment per unit length at the center, $M = P\ell$, where ℓ is the half of the plate length. The ratio of the maximum moment per length and the maximum transverse shear force per length is defined as:

$$\ell = \frac{M}{V}, \tag{2}$$

and it has dimension of length, Wicks and Hutchinson [3].

In optimization of the sandwich plate with truss core, four modes of failure are used, yielding of the face sheet, buckling of the face sheet, yielding of the truss core member and buckling of the truss core member. The corresponding constraints, respectively, are, Wicks and Hutchinson [4]:

$$\frac{M}{t_f H_c} \le \sigma_y, \tag{3}$$

$$\frac{M}{t_f H_c} \le \frac{49\pi^2 E}{432\left(1-\nu^2\right)}\left(\frac{t_f}{d}\right)^2, \tag{4}$$

$$\frac{\sqrt{3} V d L_c}{H_c \pi R_c^2} \le \sigma_y, \tag{5}$$

$$\frac{\sqrt{3} V d L_c}{H_c \pi R_c^2} \le \frac{\pi^2 E R_c^2}{4 L_c^2}, \tag{6}$$

where $d = \sqrt{L_c^2 - H_c^2}$.

The task is to minimize the mass defined by equation (1) in terms of four dimensionless geometric parameters:

$$\left(x_1, x_2, x_3, x_3\right) = \left(\frac{t_f}{\ell}, \frac{R_c}{\ell}, \frac{H_c}{\ell}, \frac{d}{\ell}\right), \tag{7}$$

for four constraints defined by equations (3) – (6). Mass, given by equation (1), in the dimensionless form is given as:

$$\frac{W}{\rho\ell} = 2\left(x_1 + \pi x_2^2 \cdot \frac{\sqrt{x_3^2 + x_4^2}}{x_4^2 \sqrt{3}}\right). \tag{8}$$

The corresponding restrictions in the dimensionless form are:

$$\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_y}\right) x_1^{-1} x_3^{-1} \le 1, \tag{9}$$

$$\left(\frac{V^2}{ME}\right)\frac{432\left(1-\nu^2\right)}{49\pi^2}\cdot\frac{x_4^2}{x_1^3 x_3}\le 1\,, \tag{10}$$

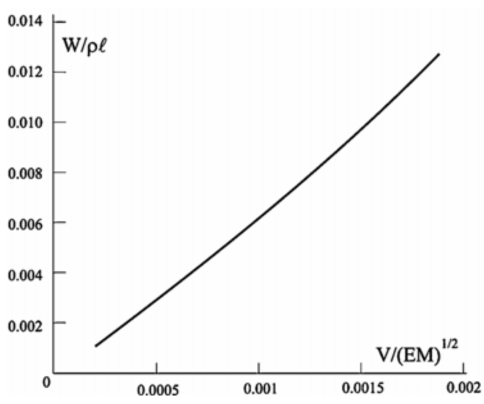
$$\frac{\sqrt{3}}{\pi}\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_y}\right)\frac{x_4\sqrt{x_3^2+x_4^2}}{x_3 x_2^2}\le 1\,, \tag{11}$$

$$\frac{4\sqrt{3}}{\pi^3}\left(\frac{V^2}{ME}\right)\frac{x_4\sqrt{\left(x_3^2+x_4^2\right)^3}}{x_3 x_2^4}\le 1\,. \tag{12}$$

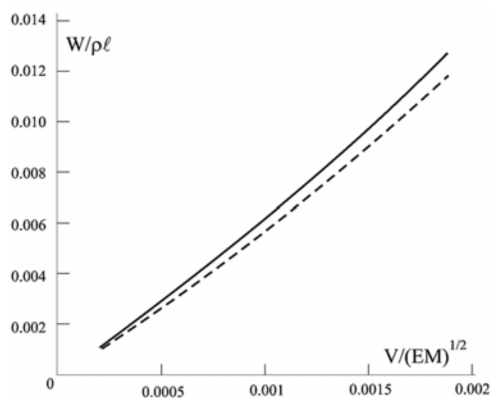
In equations (8) – (12) appear two dimensionless parameters, material dimensionless parameter $\left(\sigma_y/E\right)$ and load dimensionless parameter $\left(V^2/EM\right)$. In equation (11) also appears the Poisson's ratio for which the adopted value is $\nu=\left(1/3\right)$.

3. Results and discussion

In this paper aluminum was taken as the sandwich plate material for which the material dimensionless parameter $\left(\sigma_y/E\right)$ has the value ~ 0.007. The relation between mass in the dimensionless form and the load dimensionless parameter was obtained by the symbolic programming routine *Mathematica*® and is presented in Figure 3a.



(a)



(b)

Fig. 3 Normalized mass per unit length as a function of the dimensionless load parameter for the optimal sandwich plate with truss core (a), comparison of truss core (solid line) and sandwich plate with honeycomb core (broken line)

In Figure 3b is presented comparison of the optimal mass of the sandwich plate with truss core and the sandwich plate with the honeycomb core, for which details were obtained form Wicks and Hutchinson [4]. From this Figure, one can see that the sandwich plate with honeycomb core has the lower mass than the sandwich plate with truss core, over the whole load range, though this advantage is not very prominent, especially if the flat faces for both types of plates are of the same thickness.

The optimal values of the sandwich plate dimension parameters are presented in Figure 4. Optimization solution was also obtained by the symbolic programming routine *Mathematica*®.

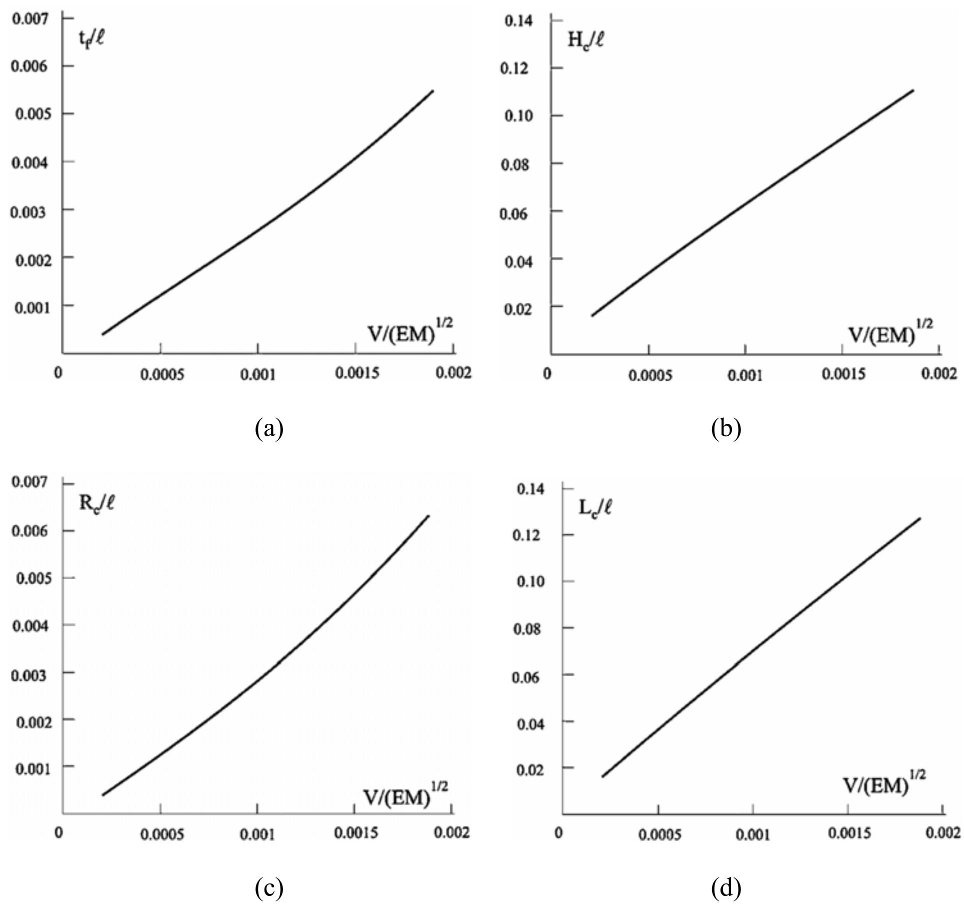


Fig. 4 Optimal: (a) thickness of the sandwich plate faces; (b) core thickness; (c) core member cross-section radius; (d) core member length

4. Conclusion

Sandwich plates with truss core are almost as efficient as the sandwich plates with honeycomb core, optimized to carry the combination of loads by the bending moment and transverse shear force. Considering that the difference in mass is very small, in selecting which type of structure should be chosen for certain construction, one should take into account other major advantages of plates with truss core, like easier manufacturing, sensitivity (proneness) to delamination and humidity, and above all multi-functionality. In all those aspects sandwich plates with truss core have advantages over plates with honeycomb core.

Nomenclature

E – Young's modulus of elasticity, H_c – truss core thickness, L_c – truss core member length, M – bending moment per unit length, R_c – core member cross-section radius, V – transverse shear force per unit volume, W – plate mass per unit area, ℓ – plate length half, t_f – face sheet thickness, x_1, x_2, x_3 – dimensionless geometric parameters, θ – angle between the core member and faces, ν – Poisson's ratio, ρ – material density, σ_y – yield strength.

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Summary

Sandwich plates with truss core have many advantages with respect to other structures, including the plates with the honeycomb cores. Besides the architectural possibilities, which are the consequence of their favorable appearance, these structures can carry equal or even higher loads than other similar structures. In this paper is presented an attempt to optimize the mass of the sandwich plate's structure, which is subjected to simultaneous action of loads due to the transversal shear force and the bending moment. In comparison to the corresponding sandwich plate with the honeycomb cores, the plate with the truss core is almost equally efficient form the aspect of the mass decrease.