ROCZNIKI INŻYNIERII BUDOWLANEJ – ZESZYT 12/2012
Komisja Inżynierii Budowlanej Oddział Polskiej Akademii Nauk w Katowicach

COMPARISON OF ANALYTICAL SOLUTIONS WITH NUMERICAL MODELING RESULTS OF CONTACT PROBLEM OF THE SHALLOW FOUNDATIONS INTERACTION WITH SUBSOIL

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1. Introduction

Numerical modeling by Finite Element Method (FEM) is often making use in solving difficulties of the engineering problems. FEM provides very universal and efficiency tool on their solution. For cases contact tasks of rigid circular and rectangular foundations loading centric force are comparison results of solution for normal stresses according to analytical methods and finite element method.

When solving geotechnical problems are most frequently used model solutions based on different theoretical assumptions and boundary conditions. All previous approaches are largely influenced by the interdisciplinary character and complexity of the geotechnical problems. In general, to solve geotechnical problems is possible in terms of mathematical precision in principle, be used three basic approaches:

- exact solutions (analytical method),
- approximate solutions (numerical, empirical, experimental methods),
- combined solution (semi-empiric methods in which the solution of the problem of sub-tasks alternated exact and approximate methods).

Suitability for use of these methods is mainly determined by the complexity of the problem and requirements for the precision of the results.

Analytical methods for providing so called "exact closed form solution" can be applied only to a small group of very simple tasks with a precise and clearly defined, usually strongly idealized, respectively with simple boundary conditions. Limited use of analytical solutions mainly caused by the fact that the construction practice are made increasingly difficult building structures in more complicated geotechnical conditions. Therefore, in the calculation should be replaced real engineering geological subsoil conditions suitable subsoil model with idealized material properties. Suitability of idealized real properties into subsoil geotechnical model has a decisive influence on the correctness and precision of solutions.

2. Boundary conditions of the contact problem

To the fundamental problems in solving of the geotechnical problems include determining changes in the rock mass stresses under action of the external loads. Correct determine the state of stress of the rock mass is a quantitative and qualitative decisive impact on the reliability of the building structure under consideration by first and second limit states. For comparison the accuracy of the results of calculations made by analytical methods and numerical results obtained by finite element method, was chosen contact task. Contact task was to determine the vertical distribution of the normal stresses in the gaps level of the rigid foundation loading by centric force. For more complete analysis were chosen three basic geometrical shapes of the shallow foundations: circular, square and rectangular, respectively strip foundation with an aspect ratio l/b ≅ 10 [1]. Geometrical parameters and stiffness of solved rigid shallow foundation models are given in Tab. 1. The foundation relative stiffness ”k” is defined according to the formula [2]:

$$k = \frac{E_f}{E_{def}} \left(\frac{t}{B}\right)^3 \quad \text{respectively} \quad k = \frac{E_f}{E_{def}} \left(\frac{t}{L}\right)^3 , \qquad (1)$$

where E_f is modulus of elasticity of foundation,
E_{def} modulus of elasticity of subsoil,
t foundation thickness ,
B, L foundation width "B", respectively length "L".

For relative stiffness k<1 the foundation is consider as a flexible and for k>1 the foundation is consider as a rigid.

The contact task is solved as a 3-D problem according to assumptions of the linear elastic half-space theory. For expression of the "foundation - subsoil" system stiffness was considered in the calculation of the foundation model from steel, which is founded on a dense sand subsoil. Physical properties of foundations and subsoil used in the calculations are given in Table 2.

Table 1 Geometrical characteristics and stiffness of the shallow foundations models

Geometrical shapes of foundations	Ratio L/B (-)	Foundation dimensions			Foundation relative stiffness STN 73 1001 [2]	Consider of relative stiffness [2]
		Width B (mm)	Length L (mm)	Thickness t (mm)	k (-)	
SQUARE	1	200	200	100	1009.6	Rigid
CIRCULAR	-	240	-	100	584.2	Rigid
STRIP	≅ 10	65	630	100	32.3 *	Rigid

* stiffness in length direction “L“ of strip foundations

Table 2 Physical properties of foundations and subsoil

Model	Material	Physical properties		
		Modulus of elasticity E (MPa)	Poisson’s ratio ν (-)	Relative density I_D (-)
Foundation	Steel	210 000	0.20	-
Subsoil	Sand	26	0.28	0.7

3. Analytical solutions

Analytical solution of the contact tasks of the rigid circular punches laid on isotropic elastic half-space and loaded with centric force was derived by Borowicka, 1939 (In Selvadurai, 1979) [3]. The solution is derived for Poisson's number $\nu = 0.5$, i.e. provided that the foundation and the subsoil are frictionless (smooth contact). Poulos and Davis [4] report for these analytical solutions calculation of the vertical contact stresses in the form:

$$\sigma_c = \frac{\sigma_m}{2 \cdot \sqrt{\left(1 - \frac{x^2}{r^2}\right)}}, \qquad (2)$$

where σ_c - the contact stress in the foundation gaps,

σ_m - average stress in the foundation gaps,

r - the radius of the circular foundation,

x - the distance from the axis of foundation.

Approximate solution of integral equations describing the relationship between elastic deformation and the stress (by developing a power type to the Maclaurin series) for rigid foundation structure of rectangular ground plan shape and provided $\nu = 0.5$ dealt Wünsch (1947) [5] . To calculate the vertical contact stress relation he derived in a closed form:

$$\sigma_c = \sigma_m \cdot \frac{4}{n(\chi)} \cdot \left[\frac{\chi}{\operatorname{arc\,sinh} \chi} \cdot \frac{1}{\sqrt{1-\xi^2}} + \frac{1}{\operatorname{arc\,sinh} 1} \cdot \frac{1}{\sqrt{1-\zeta^2}} - 1 \right], \qquad (3a)$$

where:

$$n(\chi) = \left(\frac{\chi}{\operatorname{arc\,sinh} \chi} \cdot 2\pi + \frac{1}{\operatorname{arc\,sinh} 1} \cdot 2\pi - 4 \right), \qquad (3b)$$

χ - is ratio of length to width of the foundation (the L / B = 1 apply in the case of a square foundation, respectively L / B > 1 solution applies to a rectangular foundation),

ξ, ζ - is relative distance from any point from the axis of the foundation length direction "L", respectively width foundation direction "B".

Comparing these solutions for circular and a rectangular foundation is evident that a relatively more complicated boundary conditions (applicable to the rectangular foundation) the advantage is lost in a closed form solution, that relative simplicity and mathematical "elegance", respectively clarity of solutions.

4. Finite Element Method (FEM) solution - definition of the boundary conditions

Input data and boundary conditions have been entered into geotechnical models with geometric parameters and material properties, which are listed in Table 1 and Table 2. In the model, the soil was assumed that the dense sand is placed in a steel cylinder with a diameter and height equal to 0.8m. Schematic illustration of an idealized geotechnical model for circular foundation is in Fig. 1. For comparative calculations were considered with centric load force F that causes the unit average contact stress (σ_m = 1.0 kPa). Example of meshing of the computational model for circular foundation on finite elements is shown schematically in Fig. 2.

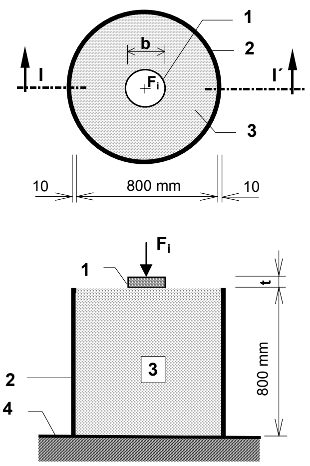

Fig. 1 View the physical model of interaction (1 - model of circular foundation, 2 - steel cylinder, 3 - sandy subsoil, 4 - solid base)

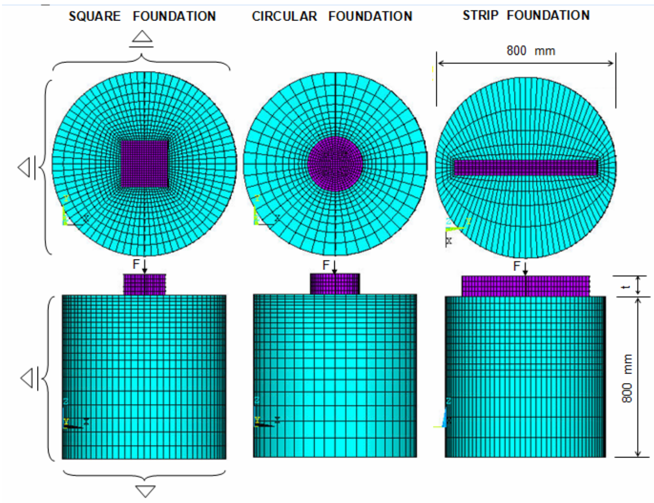

Fig. 2 Complete numerical models of rigid (square, circular and strip) foundations (thickness t = 100 mm) with static boundary conditions

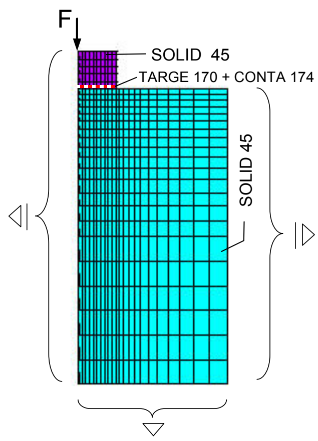

Fig. 3 Solved numerical model of rigid square foundations with one-directional bond (1/4 of axis - symmetrical model)

The task is solved by the computer program ANSYS® [6], which is based on theoretical assumptions of the Finite Element Method (FEM). For meshing continuous areas of the foundation model and subsoil were used 3D 8-node finite elements "SOLID45". An important part of the solution was to provide the same boundary conditions on the contact between the foundation and subsoil to suggest solutions in a closed form (contact without friction, respectively smooth contact without causing shear forces). Therefore, the area of contact surface was modeled with the 8-node 3-D contact elements "CONTA 174" and "TARGE 170" (Fig. 3). These elements ensure contact between adjacent nodal points of foundation and subsoil, excluding the effects of the transfer of tensile forces.

5. Evaluation of the results of calculations

Evaluation of the results of model calculations in more detail focused on the influence of division, respectively dividing details of the computational model on precision the solutions. A series of (approximately 45) model calculations for different subsoil dividing on the finite elements in the foundation level of gaps were realized. The results obtained by FEM were compared with the analytical results. The results of vertical contact stress in the axis of the foundations are shown in Table 3. Comparison of the calculated contact stress distribution according the analytical and numerical methods of solution is graphically illustrated in Fig. 4.

Table 3 Comparison of analytical and numerical (FEM) solutions for the foundation axis

Geometrical shape of foundations	Relative vertical normal contact stress σ_c / σ_m [-] Analytical solution	FEM solution	Difference [%]	The size of the finite element in the level of contact area (ratio to "B", respectively "L") Width x Length x Height
Square	-0.5000	-0.5033	0.67	0,0416(B) x 0,0416(B) x 0,0473(B)
Circular	-0.4949	-0.4895	-1.09	0,05(B) x 0,05(B) x 0,071(B)
Strip (L/B ≈ 10)	-0.5763	-0.5671	-1.59	0,013(L) x 0,025(L) x 0,053(L)

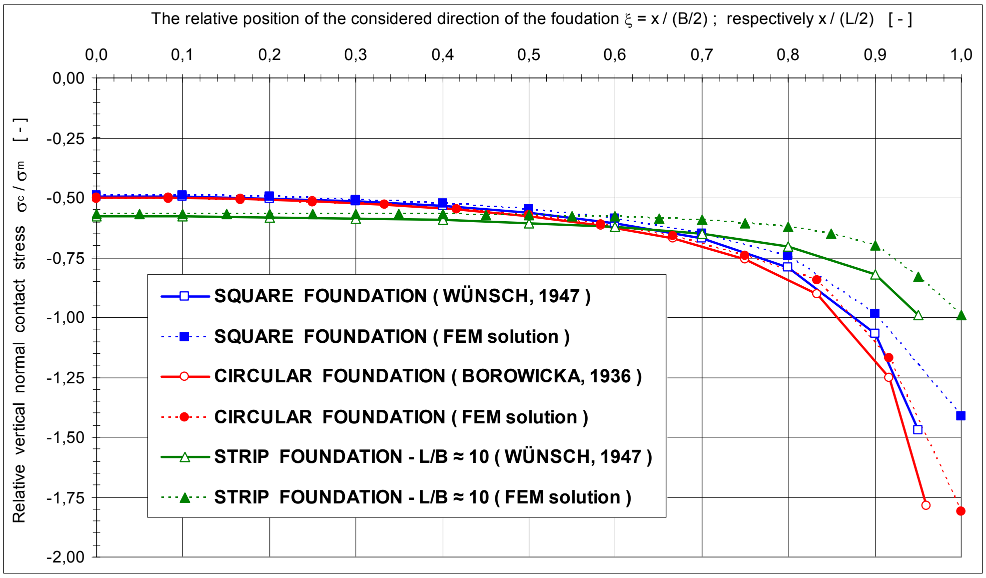

Fig. 4 Comparison of analytical and numerical (FEM) calculations of the relative contact stress distribution under rigid circular, square and rectangular foundation

From the above results follows that a sufficiently detailed meshing the area of interest in the finite element model (about 5% of the width of circular and square foundations, respectively 2 to 5% of rectangular foundations) can be achieved relatively very high precision in the calculation of contact stresses (approximately into 2 %). Following the analysis of the results obtained it can be concluded that the appropriate optimization computational model dividing into finite elements can be obtained relatively very high precision calculations in solving geotechnical problems.

6. Conclusion

In solving difficult engineering problems is increasingly being used to model solutions using mathematical Finite Element Method, which gives a very universal and effective tool to deal with them. In the process of idealization, respectively simplification of the real boundary conditions of the model is important to consider the character and purpose of the solved task. The main assumption for correct and economical solutions belong consideration of the tasks physical nature in all its phases of solutions. For precision of the results is also an important influence optimal dividend of the foundations models. In respect of the above, it is very possible to achieve relatively accurate results by using the FEM.

References

[1] Hruštinec, Ľ.: Interaction analysis of the shallow foundations with subsoil. Ph.D. Thesis, Bratislava, 2002, 689 pp. (In Slovak).
[2] STN 73 1001: Subsoil under shallow foundation. Slovak Standard, 1987, (In Slovak).
[3] Selvadurai, A.P.S.: Elastic Analysis of Soil-Foundation Interaction. Amsterdam 1979, 543 p.
[4] Poulos, H.G. - Davis, E.H.: Elastic Solutions for Soil and Rock Mechanics. 1974, 411p.
[5] Wünsch, J.: Rigid foundation and elastic half-space. Konstrukter 25, Prague 1947, 164 pp. (In Czech).
[6] ANSYS®: User's Manual, Swanson Analysis System, Inc., Volume I. - IV., 1999.

Acknowledgement

The Authors are grateful for support from the Grant Agency VEGA of the Slovak Republic, project No. 1/0629/12.

POROVNANIE ANALYTICKÉHO RIEŠENIA A VÝSLEDKOV NUMERICKÉHO MODELOVANIA KONTAKTNÉHO PROBLÉMU SPOLUPÔSOBENIA PLOŠNÝCH ZÁKLADOV S PODLOŽÍM

Zhrnutie

Pri riešení náročných inžinierskych problémov sa často využíva numerické modelovanie s využitím matematického aparátu metódy konečných prvkov (MKP). MKP poskytuje veľmi univerzálny a efektívny nástroj na ich riešenie. Pre prípady kontaktnej úlohy tuhého kruhového a pravouhlého základu zaťaženého centrickou silou sú porovnané výsledky výpočtov zvislých normálových napätí získané podľa analytických riešení a riešení MKP.