

## NUMERICAL SIMULATION OF VEHICLE MOTION ALONG THE ROAD STRUCTURE

Jozef MELCER and Gabriela LAJČÁKOVÁ  
 University of Zilina, Faculty of Civil Engineering,  
 Department of Structural mechanics

### 1. Introduction

The roads are the transport structures subjected to direct dynamic effect of moving vehicles. The knowledge of the real load acting on the roads, its variability in time and frequency composition, is needed for the solution of many engineering tasks as design, fatigue, lifetime, reliability, maintenance, structure development, micro-tremor, etc. The task can be solved by experimental or by numerical way. But the most effective way is the combination of the both mentioned advances. The submitted paper is dedicated to the description of facilities how to obtain the required data by numerical way. This process requires creation of the computing models of vehicles, the computing models of the roads and to pay attention to the numerical solution of equations of motion and the analysis of obtained results in time and in frequency domain.

### 2. Vehicle computing model

The computing models of vehicles can be created on various levels as 1-dimensional, 2-dimensional or 3-dimensional [1]. For the purpose of this contribution the 3-dimensional space computing model of a lorry was adopted, Fig. 1.

It is discrete computing model with 15 degrees of freedom. The 9 mass degrees of freedom correspond to the vertical displacements  $r_i(t)$  of the mass objects  $m_i$ . The mass-less degrees of freedom correspond to the vertical movements of the contact point of the model with the road surface. The vibration of the mass objects of the model is described by the 9 functions of time  $r_i(t)$ , ( $i = 1 \div 9$ ). The mass-less degrees of freedom are associated with the tire forces  $F_j(t)$ , ( $j = 10 \div 15$ ) acting at the contact points. The equations of motion have the form of ordinary differential equations and with respect on the used method of numerical solution they can be written in the form

$$[m] \cdot \{\ddot{r}\} + [b] \cdot \{\dot{r}\} + [k] \cdot \{r\} = \{F\} , \quad (1)$$

$$[m] \cdot \{\ddot{r}\} = \{F\} - [b] \cdot \{\dot{r}\} - [k] \cdot \{r\} = \{F\} - \{F_d\} - \{F_{re}\} = \{F_v\} . \quad (2)$$

Solution of equations of motion is realized numerically in the environment of programming system MATLAB.

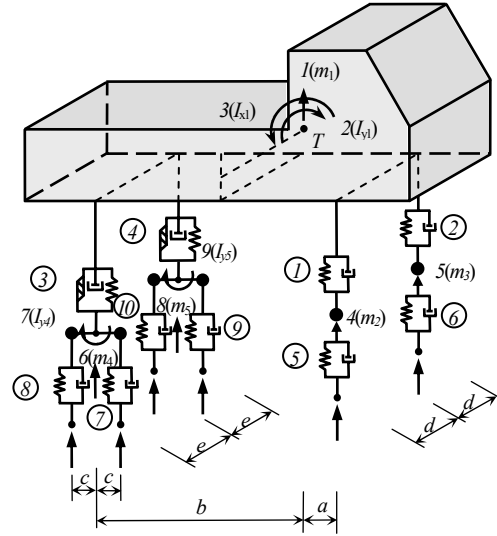


Fig. 1 Space model of the vehicle TATRA 815

### 3. Road surface unevenness

The rigid pavement with random road profile is assumed for the purpose of numerical solution. The random road profile  $h(x)$  is assumed as stationary ergodic function with zero mean value and normal distribution. The properties of the road profile are described by Power Spectral Density function (PSD) in the form

$$S_h(\Omega) = S_h(\Omega_0) \cdot \left( \frac{\Omega}{\Omega_0} \right)^{-k}, \quad (3)$$

where  $\Omega$  in [rad/m] denotes the wave number,  $\Omega_0 = 1$  rad/m is the reference wave number and the waviness  $k = 2$ . According to the international directive ISO 8608 [2], typical road profiles can be grouped into classes from A to E. Each class is simply defined by its reference value  $S_h(\Omega_0)$ , Fig. 2, Table 1.

Table 1: Classification of pavements according to road unevenness [2]

Class	$S_h(\Omega_0)$ [ $\text{m}^2/(\text{rad}/\text{m})$ ] at $\Omega_0 = 1$ rad/m		
	lower bound	geometric average	upper bound
A	-	$1 \cdot 10^{-6}$	$2 \cdot 10^{-6}$
B	$2 \cdot 10^{-6}$	$4 \cdot 10^{-6}$	$8 \cdot 10^{-6}$
C	$8 \cdot 10^{-6}$	$16 \cdot 10^{-6}$	$32 \cdot 10^{-6}$
D	$32 \cdot 10^{-6}$	$64 \cdot 10^{-6}$	$128 \cdot 10^{-6}$
E	$128 \cdot 10^{-6}$	$256 \cdot 10^{-6}$	$512 \cdot 10^{-6}$

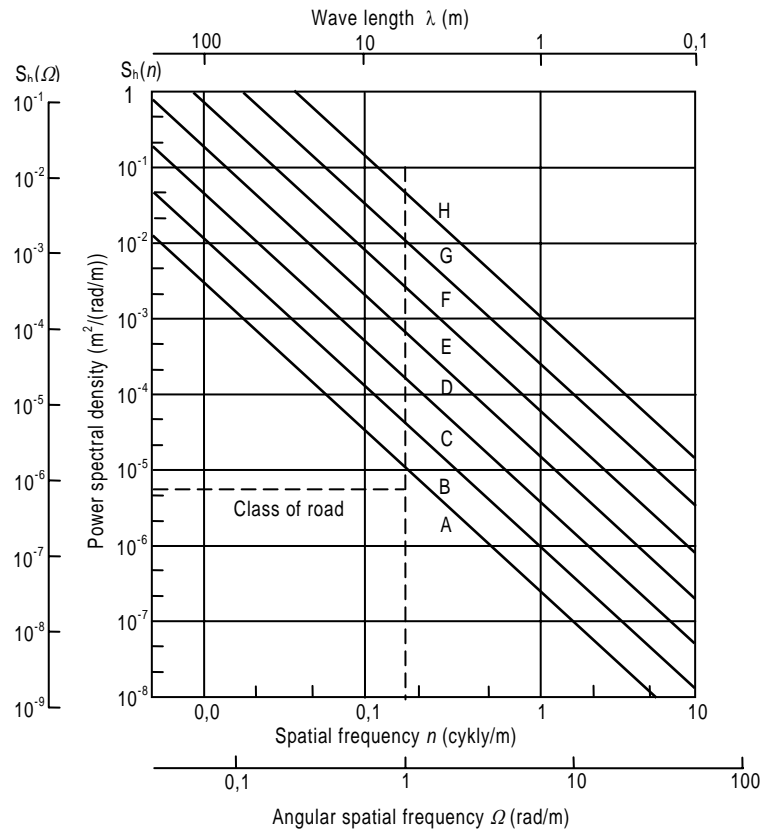


Fig. 2 Power spectral densities of road profiles

A random profile of a single track can be approximated as

$$h(x) = \sum_i^N \sqrt{2 \cdot S(\Omega_i) \cdot \Delta\Omega} \cdot \cos(\Omega_i \cdot x + \varphi_i), \quad (4)$$

where  $\varphi_i$  is the uniformly distributed phase angle.

#### 4. Results of numerical simulation of the tire forces

Numerical solution was carried out in the environment of the program system MATLAB. The 4<sup>st</sup> order **Runge-Kutta** step-by-step integration method was used for the solution of equations of motion [3]. As the result of numerical simulation the movement of vehicle along the road profile with the speed  $V = 36$  km/h was simulated. In the first step the random road profile  $h(x)$  on the basis of known power spectral density for the value  $S_h(\Omega_0) = 4 \cdot 10^{-6}$  m<sup>2</sup>/(rad/m), category B, was generated. The generated road profile is shown in Fig. 3.

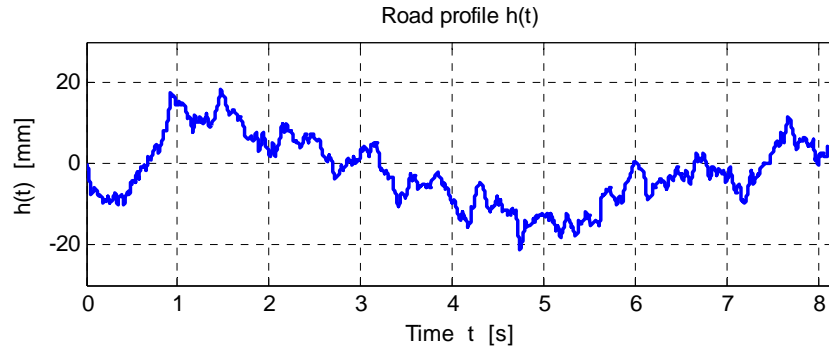


Fig. 3 Random road profile,  $S_h(\Omega_0) = 4 \cdot 10^{-6} \text{ m}^2/(\text{rad/m})$

The corresponding tire forces under rear wheel of rear axle of the vehicle computing model are shown in Fig. 4.

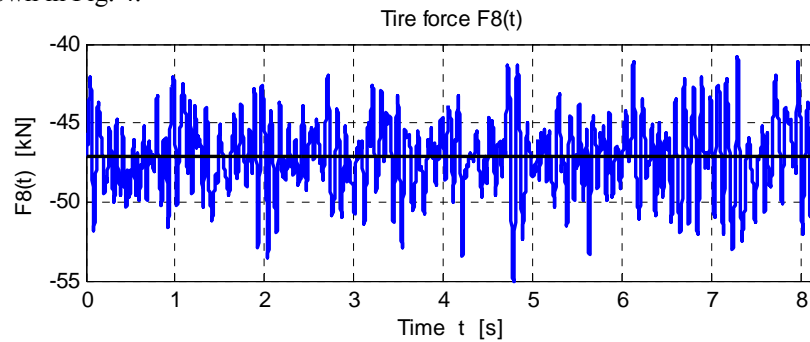


Fig. 4 Tire forces under rear wheel of rear axle

Similar results can be obtained for various road profiles, for various speeds of vehicle motion and for various vehicle computing models. At the certain category of road the tire forces are theoretically positive. In reality the contact between the wheel and the road is lost. The wheel will be bounce from the road. The contact is always replaced by the impact. It is very bad situation not only for the pavement but also for the vehicle.

The solution can be carried out in the time or in the frequency domain. In the frequency domain the frequency composition of vibration is interesting. It can be assessed for example through Power Spectral Density functions (PSD). The PSD function of the road profile informs us about the frequency composition of the road profile and the PSD of the tire force informs us about the frequency composition of the pavement load. The PSD of the road profile from Fig. 3 is plotted in Fig. 5 and the PSD of the dynamic component of tire force from Fig. 4 is plotted in Fig. 6. As we can see in the frequency composition of the road profile the low frequencies dominate. In the frequency composition of the tire forces the frequencies in the interval from 6 to 12 Hz dominate. It relates with natural frequencies of the vehicle.

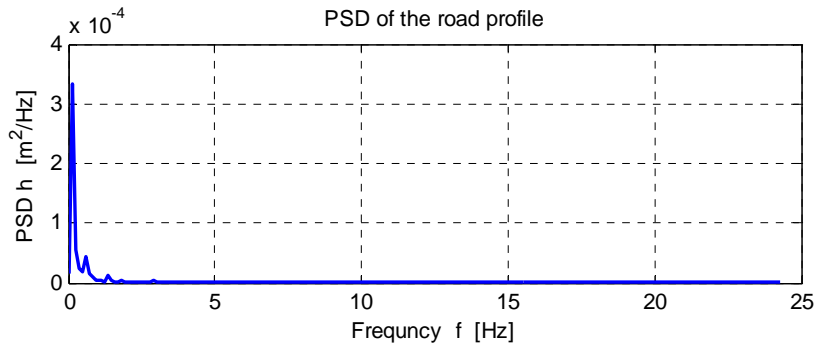
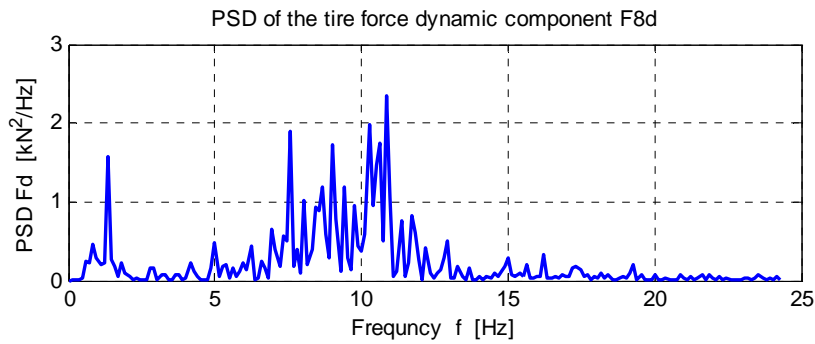
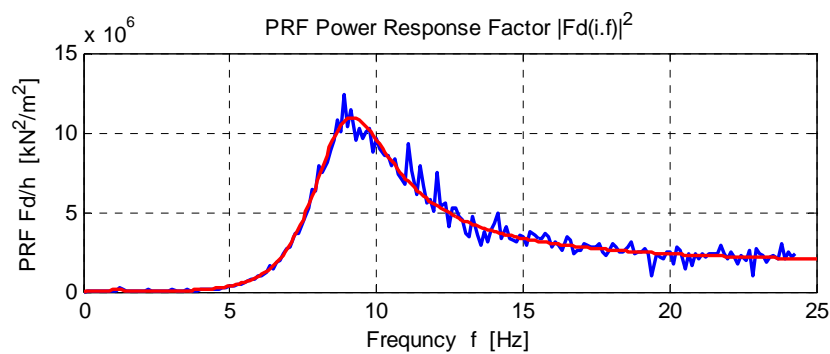


Fig. 5 PSD of the road profile

Fig. 6 PSD of tire force  $F_{8d}(t)$ 

Road profile  $h(t)$  represents the input function and tire force represents the output function. The Frequency Response Function (FRF) between the two quantities can be derived. The second power of the modulus of FRF is denoted as Power Response Factor (PRF). It can be obtained by dividing the PSD of output and input functions.

Fig. 7 Power response factor of the tire force  $|F_{8d}(i.f)|^2$ 

Smoothing of PRF obtained by dividing the PSD of output and input functions (red line) we receive the same results as at direct solution in frequency domain for example by Fourier or Laplace integral transform.

#### 4. Conclusions

Numerical modeling of the problems of vehicle - road interaction is an effective tool for the solution of real tasks of engineering practice. Quality of obtained results is dependent on the quality of input data. The present state of computing technique enables the numerical processing of solved problems in real time. From the practical point of view the influence of road profile on vehicle vibration and tire forces is interested. The results obtained from numerical analyses are used in the process of design and optimization of road structures with respect to lifetime, reliability and environmental aspects.

#### References

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- [2] ISO 8608, *Mechanical vibration – road surface profiles – reporting of measured data*. International standard, 1995.
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### NUMERICKÁ SIMULÁCIA POHYBU VOZIDLA PO CESTNÝCH KOMUNIKÁCIÁCH

#### Resumé

Predkladaný príspevok je venovaný metódam numerického modelovania pohybu vozidiel po cestných komunikáciách. Uvažuje priestorový výpočtový model vozidla TATRA 815 a výpočtový model tuhej vozovky s náhodným profilom povrchu jazdnej dráhy. Prezentované je numerické riešenie pohybu vozidla po vozovke v časovej i vo frekvenčnej oblasti. Riešenie pohybových rovníc je robené v prostredí programovacieho jazyka vyššej úrovne MATLAB. Získané výsledky sú zobrazené v grafickej forme.