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OPTIMAL PLATES WITH PRISMATIC CORES

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1. Introduction

Sandwich plates with prismatic cores have open structure, high stiffness and strength; thus they have wide application. Allen [1] has investigated their characteristics. Evans et al. [2] have examined the mechanical and thermal properties of cellular metal systems, comparing them to other dense materials. They have done some design analyses for prototypical systems, which specify implementation opportunities relative to other concepts. Brittain et al. [3] have estimated the mechanical performance and structural efficiency of the truss beam under the four-point bending and compared to the bending behavior of a square box beam. Wicks and Hutchinson [4] have optimally designed for minimum weight sandwich plates, comprised of truss cores faced with either planar trusses or solid sheets, subjected to prescribed combinations of bending and transversal shear loads. Wicks and Hutchinson [5] have optimized the weight of truss core sandwich plates, subjected to a crushing stress and arbitrary combinations of bending and transversal shear subjected to buckling and plastic yielding constraints and compared them to the weight performance of other types of optimized plates. Valdevit et al. [6] have analyzed multifunctional sandwich panels with corrugated and prismatic double diamond cores. They compared their behavior to panels designed using truss and honeycomb cores. Plates were evaluated for optimal dimensions and the minimum weight. They also devised the failure mechanism maps that account for interactions between the core and the face members during buckling.

Well designed and manufactured construction, consisting of a plate with prismatic core, can be very efficient, from the aspect of smaller mass, what is presented in this paper.

In the case of flat plates, subjected to bending, the sandwich plates with the honeycomb cores are irreplaceable. The situation is significantly different if the plates were curved.

Then the sandwich plates with the prismatic cores become more competitive, because the plates with the honeycomb cores are more sensitive to deviations.

Structures made of plates with prismatic cores can simultaneously be used as the carrying and the heat transferring elements. Their cavities can be used as liquid storage or the pressure vessels. Other types of structures comprised of plates with the honeycomb cores or the common truss structures do not possess any of those possibilities.

2. Problem formulation

The sandwich plate with prismatic cores is presented in Fig. 1. Here an assumption was made that both the plate and the cores are made of the same material, though more efficient structure can be obtained if different materials were adopted for the plate and its cores. Material properties are: Young's elasticity modulus E, Poisson's ratio ν, yield strength σ_y and material density ρ.

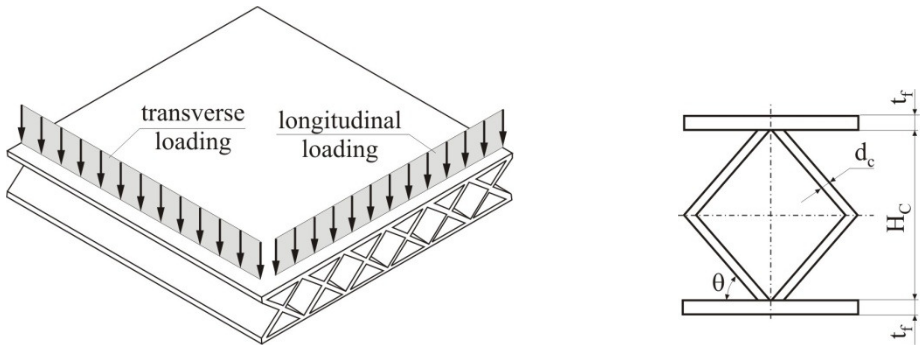

Fig. 1. The sandwich plate with prismatic cores: loading and geometry.

Mass per unit area of the sandwich plate with prismatic core, presented in Fig. 1, is:

$$W = 2\rho\left[t_f + \frac{d_C}{\cos\theta} \right]. \tag{1}$$

In general case of the infinitely wide plate, (Fig. 1), it is subjected to maximal moment per unit length M and maximal transversal shear force per unit length V. Almost total shear force is carried by the cores, while the moment is carried by the face sheets of a plate. Wicks and Hutchinson [5] defined the ratio of maximal moment per unit length to maximal transversal force per unit length as:

$$\ell \equiv \frac{M}{V}, \tag{2}$$

which has dimension of length.

During the sandwich plates' optimization four modes of failure were used:

- yielding of a face sheet,
- buckling of a flat surface of a plate,
- yielding of a cores' member and

- buckling of a cores' member.

The corresponding restrictions for the sandwich plate with prismatic cores, loaded in the transversal direction, respectively are, Valdevit et al. [6]:

$$
\begin{aligned}
&\frac{M}{2t_f d\sin\theta} \le \sigma_Y\,, \quad \frac{M}{2t_f d\sin\theta} \le \frac{k_f \pi^2 E}{48\cos^2\theta}\left(\frac{t_f}{d}\right)^2, \\
&\frac{V}{2d_C\cos\theta} \le \sigma_Y\,, \quad \frac{V}{2d_C\cos\theta} \le \frac{17\pi^2 E}{96}\left(\frac{d_C}{d}\right)^2,
\end{aligned}
\tag{3}
$$

while the corresponding restrictions for the sandwich plate with prismatic cores, loaded in the longitudinal direction, respectively are, Valdevit et al. [6]:

$$
\begin{aligned}
&\frac{M}{d(3t_f\cos\theta + d_C)\tan\theta} \le \sigma_Y\,, \quad \frac{M}{d(3t_f\cos\theta + d_C)\tan\theta} \le \frac{\pi^2 E}{18(1-\nu^2)\cos^2\theta}\left(\frac{t_f}{d}\right)^2, \\
&\frac{V}{(3t_f\cos\theta + d_C)\tan\theta} \le \sigma_Y\,, \quad \frac{V}{(3t_f\cos\theta + d_C)\tan\theta} \le \frac{\pi^2 E}{3(1-\nu^2)\sin\theta}\left(\frac{d_C}{d}\right)^2,
\end{aligned}
\tag{4}
$$

where: $d = (H_C + t_f)/2\sin\theta$ and $k_f = \left(\frac{2.4\cos\theta(d_C/t_f)^3 + 1}{1.2\cos\theta(d_C/t_f)^3 + 1}\right)^2$.

The task is to minimize the mass defined by equation (1) in terms of four dimensionless geometric parameters $(t_f/\ell,\ d_C/\ell,\ H_C/\ell,\ d/\ell)$, for four restrictions defined by equations (3) and (4). The mass defined by equation (1), expressed in the dimensionless form reads:

$$
\frac{W}{\rho\ell} = 2\left[\frac{t_f}{\ell} + \frac{1}{\cos\theta}\frac{d_C}{\ell}\right].
\tag{5}
$$

The corresponding restrictions for the load in the transversal direction, in the dimensionless form, are:

$$
\begin{aligned}
&\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_Y}\right)\frac{1}{2\sin\theta}\left(\frac{t_f}{\ell}\right)^{-1}\left(\frac{d}{\ell}\right)^{-1} \le 1, \quad \left(\frac{V^2}{ME}\right)\frac{48}{17\pi^2\sin\theta}\left(\frac{d_C}{\ell}\right)^{-3}\left(\frac{d}{\ell}\right)^{2} \le 1, \\
&\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_Y}\right)\frac{1}{2\sin\theta}\left(\frac{d_C}{\ell}\right)^{-1} \le 1, \qquad \left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_Y}\right)\frac{1}{2\sin\theta}\left(\frac{d_C}{\ell}\right)^{-1} \le 1,
\end{aligned}
\tag{6}
$$

while the corresponding restrictions for the load in the longitudinal direction, in the dimensionless form, are:

$$\begin{aligned}
&\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_Y}\right)\frac{1}{\tan\theta}\left[3\left(\frac{t_f}{\ell}\right)\cos\theta+\left(\frac{d_C}{\ell}\right)\right]^{-1}\left(\frac{d}{\ell}\right)^{-1}\leq 1,\\
&\left(\frac{V^2}{ME}\right)\frac{18(1-\nu^2)\cos^2\theta}{\pi^2\tan\theta}\left[3\left(\frac{t_f}{\ell}\right)\cos\theta+\left(\frac{d_C}{\ell}\right)\right]^{-1}\left(\frac{d}{\ell}\right)\left(\frac{t_f}{\ell}\right)^{-2}\leq 1,\\
&\left(\frac{V^2}{ME}\right)\left(\frac{E}{\sigma_Y}\right)\frac{1}{\tan\theta}\left[3\left(\frac{t_f}{\ell}\right)\cos\theta+\left(\frac{d_C}{\ell}\right)\right]^{-1}\leq 1,\\
&\left(\frac{V^2}{ME}\right)\frac{3(1-\nu^2)\cos\theta}{\pi^2}\left[3\left(\frac{t_f}{\ell}\right)\cos\theta+\left(\frac{d_C}{\ell}\right)\right]^{-1}\left(\frac{d}{\ell}\right)^{2}\left(\frac{d_C}{\ell}\right)^{-2}\leq 1.
\end{aligned} \tag{7}$$

In equations (6) and (7) to appear one dimensionless material parameter, σ_Y/E and one dimensionless load parameter $V^2/(EM)$. In the second equation of (7) also appears the Poisson's ratio for which is adopted the value of $\nu = 1/3$.

3. Results and discussion

For the sandwich plate material here is adopted aluminum, for which the dimensionless material parameter is $\sigma_Y/E \cong 0.007$. The optimization problem is solved numerically, by application of the symbolic programming package *Mathematica*®. The variation of mass, in the dimensionless form, $W/\rho\ell$ in terms of dimensionless load parameter, $V^2/(EM)$, is presented in Fig. 2, for the sandwich plate with prismatic cores loaded in lateral and longitudinal direction. The optimal mass of the sandwich plate with the honeycomb cores is also presented for the sake of comparison. Details of analysis of that sandwich plate were taken from Wicks and Hutchinson [4].

From Fig. 2 one can see that the sandwich plate with the prismatic cores has the smallest mass across the whole load range, though that advantage is not as prominent and it vanishes with load increase.

The optimal values of thickness of the isotropic plates that make the face sheets of the sandwich plate with the prismatic cores, t_f, as a function of the dimensionless load parameter $V^2/(EM)$ for the loading in the transversal and longitudinal direction are shown in Fig. 3.

From Fig. 3 can be seen that for the same load, thickness of the face sheets for the sandwich plate with the prismatic cores is lower for the longitudinal direction of loading.

The optimal values of the cores' height, H_c, in terms of dimensionless load parameter $V^2/(EM)$ for the loading in the transversal and longitudinal direction are shown in Fig. 4, while in Fig. 5 are presented optimal values of the cores thickness d_c.

From Figs. 4 and 5 can be seen that for the same load the core height, H_c and thickness d_c are also lower for the longitudinal direction of loading.

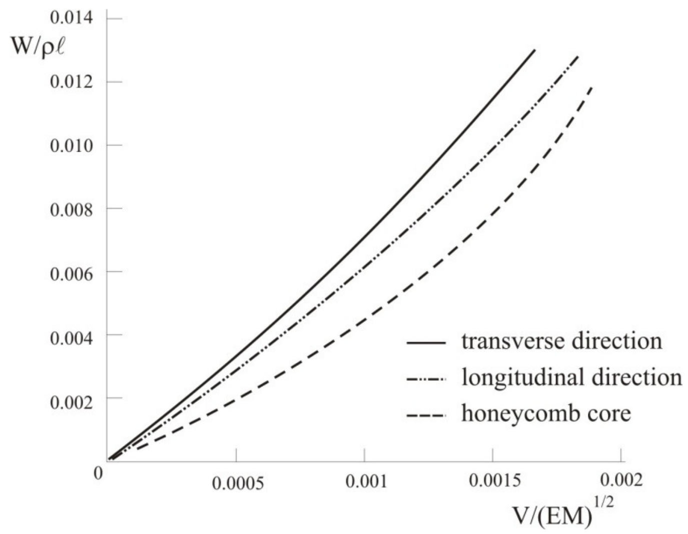

Fig. 2. Normalized mass per unit length as a function of the dimensionless load parameter for the optimal sandwich plate with the prismatic cores loaded in the transversal direction (solid line) and in the longitudinal direction (dot-dashed line) and for the sandwich plate with the honeycomb cores (broken line).

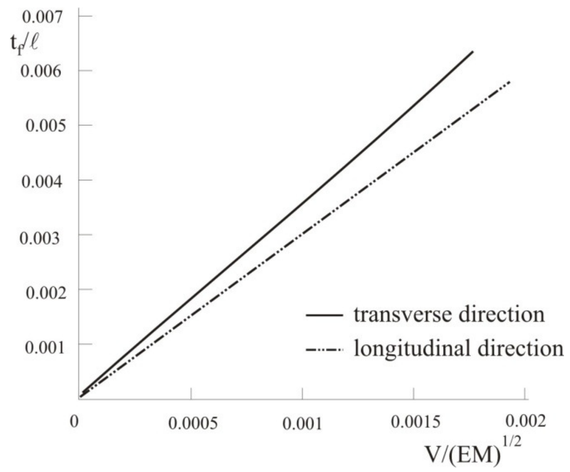

Fig. 3. The optimal thickness of the face sheets for the sandwich plate with the prismatic cores loaded in the transversal direction (solid line) and in the longitudinal direction (dot-dashed line).

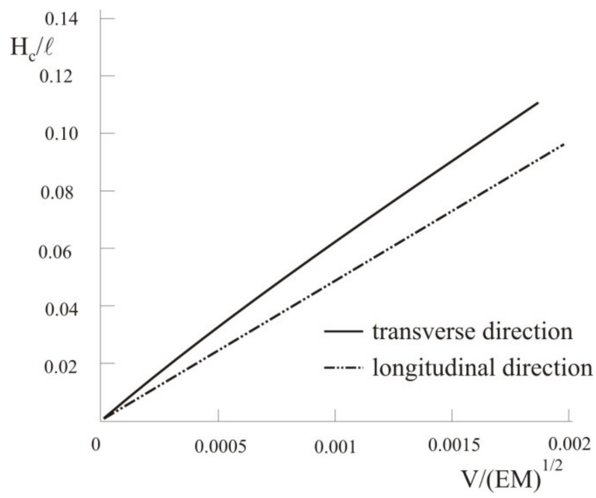

Fig. 4. The optimal cores height for the sandwich plate with the prismatic cores loaded in the transversal direction (solid line) and in the longitudinal direction (dot-dashed line)

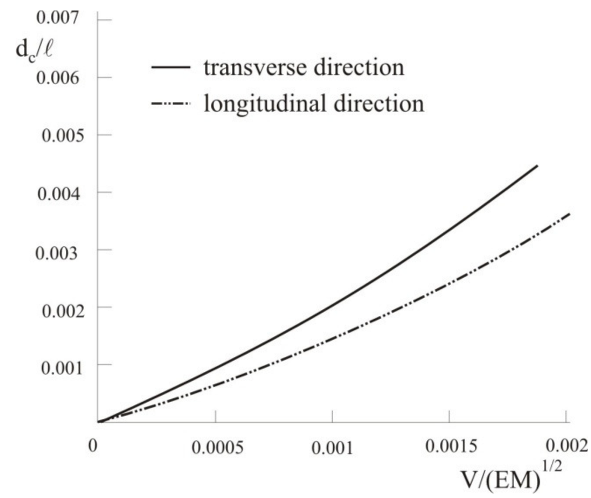

Fig. 5. The optimal cores thickness for the sandwich plate with the prismatic cores loaded in the transversal direction (solid line) and in the longitudinal direction (dot-dashed line).

From Figs. 3 to 5 one can notice that optimal dimensions can significantly differ for different loading directions. Thus, the structure made of the sandwich plate with the prismatic cores, optimized so that its mass is minimal for the given load, would not be optimal for if the load is applied in the transverse direction. The sandwich plates with the prismatic cores are thus the real option for application when the combination of carrying capacity and cooling is needed. The optimized dimensions can differ significantly, implying that the specific sandwich plates designed for minimum weight in one load direction will not perform as well when loaded in the orthogonal direction.

4. Conclusion

Plates with the honeycomb cores are more efficient mass wise, with respect to plates with the prismatic cores for the lower loads. The advantage diminishes with the load increase. The sandwich plates with prismatic cores have better performances when loaded laterally, because their characteristics are restricted by buckling of a plate, not of a beam.

Large plastic deformations of materials, which are being used in manufacturing the sandwich plates, exhibit better performances for the sandwich plates with prismatic cores.

The sandwich plates with prismatic cores are almost as efficient as the sandwich plates with honeycomb cores, when optimally designed to carry the combination of loads due to bending moment and transversal force. Since the difference in masses is very small, other factors become more significant, like easier manufacturing, sensitivity to delamination and moisture, multifunctionality, and in all those segments the plates with the prismatic cores have advantages with respect to plates with the honeycomb cores. Similar conclusions were drawn when the plates with honeycomb cores were compared to plates with truss cores [7]. The latter ones have the similar advantages with respect to the former ones, as the plates with the prismatic cores.

Nomenclature

E – Young's modulus of elasticity,
d_c – the core plate thickness,
H_c – the core thickness,
M – bending moment per unit length,
V – transverse shear force per unit length,
W – plate mass per unit area,
ℓ – plate length half,
t_f – the thickness of the isotropic plates, which make the face sheets of the sandwich plate,
θ – the angle between the core plate and horizontal (its value is taken to be $\theta = 54.7°$, because for that particular value, the shear stiffness has the largest value, Lu [7]),
ν – Poisson's ratio,
ρ – material density,
σ_y – yield strength.

Acknowledgement

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OPTIMAL PLATES WITH PRISMATIC CORES

Summary

In this paper the multifunctional sandwich plates with prismatic cores were optimized with respect to their dimensions and mass. The comparison of those plates to plates with honeycomb cores was performed. The two loading directions, longitudinal and transversal, were considered. The plates with prismatic cores have best performances when loaded longitudinally, since the plate characteristics are restricted by buckling of a plate and not of a beam.

Plates with the honeycomb cores are more efficient related to weight, than the plates with the prismatic cores for smaller loads. This advantage diminishes with load increase.

Based on this analysis a conclusion can be drawn that the sandwich plates with the prismatic cores are the most efficient from the aspect of the optimal mass, for structures which are simultaneously used for load carrying and cooling.