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VEHICLE RESPONSE IN FREQUENCY DOMAIN

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1. Introduction

The roads are the transport structures subjected to direct dynamic effect of moving vehicles. The knowledge of the real load acting on the roads and vehicle behaviour, the variability in time and frequency composition, is needed for the solution of many engineering tasks as design, fatigue, lifetime, reliability, maintenance, structure development, micro-tremor, etc [1], [2], [3]. The task can be solved by experimental or by numerical way. But the most effective way is the combination of the both mentioned advances. The submitted paper is dedicated to the description of facilities how to obtained the required data by numerical way in frequency domain. This process requires creation the computing models of vehicles, the computing models of the roads and to pay attention to the numerical solution of equations of motion in time domain and calculation of frequency spectra.

2. Vehicle computing model

The computing models of vehicles can be created on various levels as 1-dimensional, 2-dimensional or 3-dimensional. For the purpose of this contribution the 3-dimensional space computing model of a lorry Tatra 815 was adopted, Fig. 1.

It is discrete computing model with 15 degrees of freedom. The 9 mass degrees of freedom correspond to the vertical displacements $r_i(t)$ of the mass objects m_i. The mass-less degrees of freedom correspond to the vertical movements of the contact point of the model with the road surface. The vibration of the mass objects of the model is described by the 9 functions of time $r_i(t)$, ($i = 1 \div 9$). The mass-less degrees of freedom are associated with the tire forces $F_j(t)$, ($j = 10 \div 15$) acting at the contact points. The equations of motion have the form of ordinary differential equations and with respect on the used method of numerical solution they can be written in the form [4]

$$[m]\cdot\{\ddot{r}\} = \{F\} - [b]\cdot\{\dot{r}\} - [k]\cdot\{r\} = \{F\} - \{F_d\} - \{F_{re}\} = \{F_v\}\,. \tag{1}$$

Solution of equations of motion in time domain is realized numerically in the environment of programming system MATLAB. The 4th order Runge-Kutta step-by-step integration method is employed.

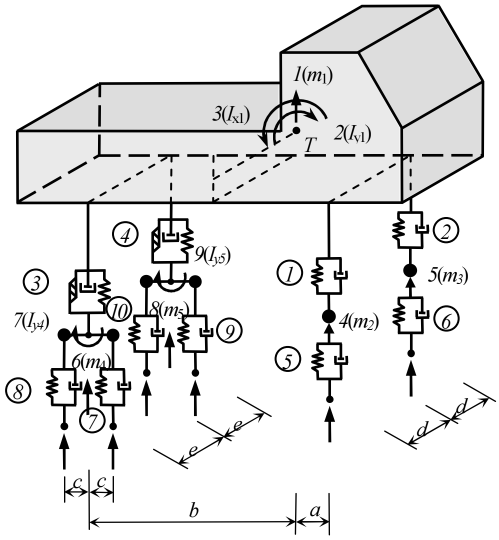

Fig. 1 Space model of the vehicle Tatra 815

3. Road unevenness

The rigid pavement with random road profile is assumed for the purpose of numerical solution. The random road profile $h(x)$ is assumed as stationary ergodic function with zero mean value and normal distribution. The properties of the road profile are described by Power Spectral Density function (PSD) in the form

$$PSD_h(\Omega) = PSD_h(\Omega_0) \cdot \left(\frac{\Omega}{\Omega_0} \right)^{-k} , \qquad (2)$$

where Ω in [rad/m] denotes the wave number, $\Omega_0 = 1$ rad/m is the reference wave number and the waviness $k = 2$. According to the international directive ISO 8608 [5], typical road profiles can be grouped into classes from A to E. Each class is simply defined by its reference value $PSD_h(\Omega_0)$, Table 1.

Table 1 Classification of pavements according to road unevenness [2]

Class	$PSD_h(\Omega_0)$ [m^2/(rad/m)] at $\Omega_0 = 1$ rad/m		
	lower bound	geometric average	upper bound
A	-	1.10^{-6}	2.10^{-6}
B	2.10^{-6}	4.10^{-6}	8.10^{-6}
C	8.10^{-6}	16.10^{-6}	32.10^{-6}
D	32.10^{-6}	64.10^{-6}	128.10^{-6}
E	128.10^{-6}	256.10^{-6}	512.10^{-6}

A random profile of a single track can be approximated as

$$h(x)=\sum_{i}^{N}\sqrt{2\cdot PSD_h(\Omega_i)\cdot\Delta\Omega}\cdot\cos(\Omega_i\cdot x+\varphi_i)\,, \tag{3}$$

where φ_i is the uniformly distributed phase angle.

4. Spectral characteristics

The Fast Fourier Transform (FFT) is used for transfer from time to frequency domain. $X(k)$ is a Fourier picture of a corresponding real number $x(k)$ obtained by the relation

$$X(k)=\sum_{n=1}^{N}x(n)\cdot\mathrm{e}^{2\pi\mathrm{i}(k-1)(n-1)/N}\text{ , for }1\le k\le N. \tag{4}$$

Fourier picture $X(k)$ is the complex number. The signal must have $N = 2^n$ samples. The sapling frequency f_S by the Shanon – Koletnik theorem should be minimally 2,5 multiple of the highest frequency inherent in frequency spectra. To every sampling number I ($I = 0 \div (N/2\text{-}1)$) the frequency value f_I is assigned by the relation

$$f_I=I\cdot\Delta f=I\cdot f_S/N \tag{5}$$

In frequency domain we usually can observe following spectral values:
Peak Amplitude

$$PA(I)=\sqrt{(\mathrm{Re}(X(I))\cdot\mathrm{Re}(X(I))+\mathrm{Im}(X(I))\cdot\mathrm{Im}(X(I)))}\,/\,N\,,\quad\text{for } I=0$$

$$PA(I)=\sqrt{(\mathrm{Re}(X(I))\cdot\mathrm{Re}(X(I))+\mathrm{Im}(X(I))\cdot\mathrm{Im}(X(I)))}\,/\,N\,,\quad\text{for } I=1\div(N/2\text{-}1)\,. \tag{6}$$

RMS Amplitude

$$RMS(I)=PA(I)/\sqrt{2}\,,\quad\text{for } I=0\div(N/2\text{-}1). \tag{7}$$

Auto Spectrum

$$AS(I)=PA(I)\cdot PA(I)\,,\quad\text{for } I=0\div(N/2\text{-}1). \tag{8}$$

Power Spectrum

$$PS(I)=AS(I)/2\,,\quad\text{for } I=0\div(N/2\text{-}1). \tag{9}$$

Power Spectral Density

$$PSD(I)=PS(I)/\Delta f\,,\quad\text{for } I=0\div(N/2\text{-}1). \tag{10}$$

Phase Spectrum

$$FS(I)=\mathrm{arctg}(\mathrm{Im}(X(I))/\mathrm{Re}(X(I)))\,,\quad\text{for } I=0\div(N/2\text{-}1). \tag{11}$$

The signal is usually modified by a weighting function. The Hanning weighting function is usually used.

$$W(I)=0{,}5\cdot(1-\cos(2\cdot\pi\cdot I/(N-1)))\,,\ \text{for } I=0\div(N\text{-}1). \tag{12}$$

5. Numerical results

The passing of vehicle along road unevenness (class B), the same in the right and left track, with the speed 36 km/h was numerically simulated in time domain. All the results were transformed by FFT from time to frequency domain. The Power Spectral Density (PSD) was calculated for the monitored quantities. The PSD of road unevenness under right front wheel is plotted in the Fig. 2. The PSD of vertical displacement of vehicle gravity centre is plotted in the Fig. 3. The PSD of vertical displacement of right front wheel is plotted in the Fig. 4.

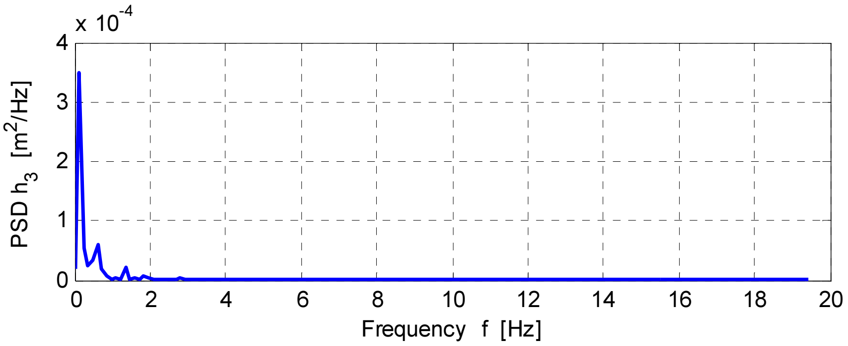

Fig. 2 PSD of road unevenness under right front wheel

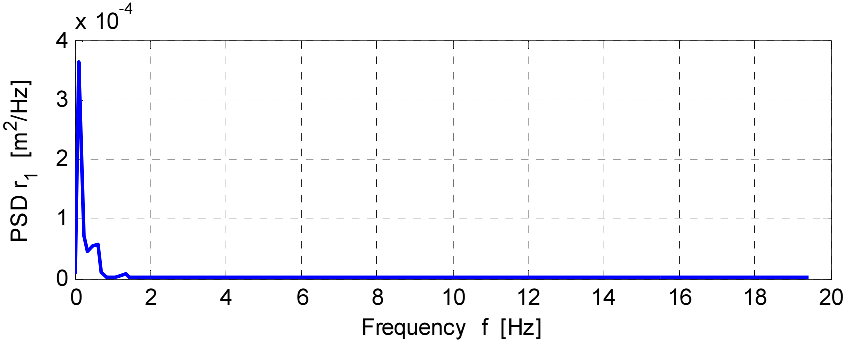

Fig. 3 PSD of vertical displacement of vehicle gravity centre

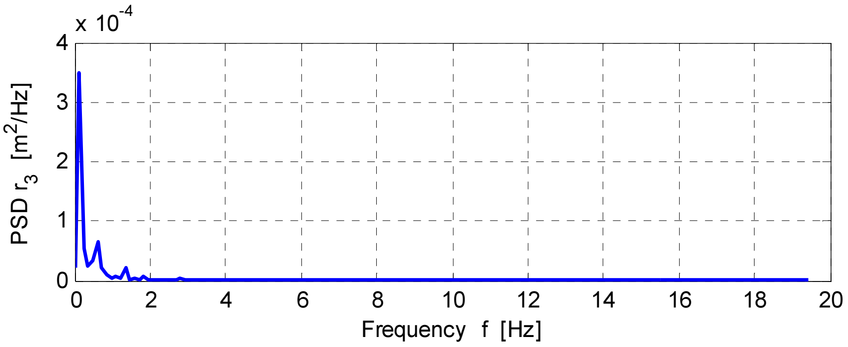

Fig. 4 PSD of vertical displacement of right front wheel

The PSD of dynamic component of tire force under right front wheel is plotted in the Fig. 5.

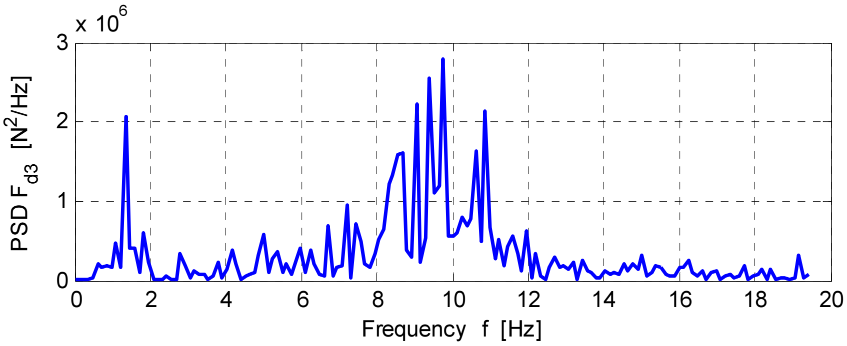

Fig. 5 PSD of dynamic component of tire force under right front wheel

The PSD of the response PSD_r and PDS of excitation PSD_e are mutually coupled by the relation

$$PSD_r(f) = \left|H(\mathrm{i}f)\right|^2 \cdot PSD_e(f)\,, \tag{13}$$

where $\left|H(\mathrm{i}f)\right|^2$ is so called Power Response Factor (PRF). The PRF of dynamic component of tire force under right front wheel is plotted in the Fig. 6

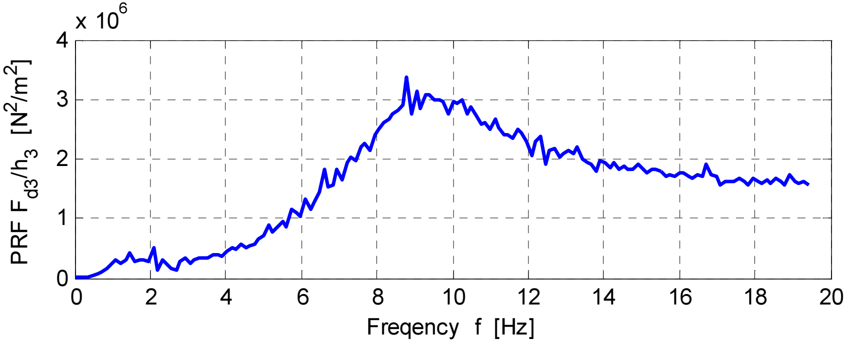

Fig. 6 PRF of dynamic component of tire force under right front wheel

6. Conclusions

Numerical modeling of the problems of vehicle - road interaction is an effective tool for the solution of real tasks of engineering practice. The present state of computing technique enables the numerical processing of solved problems in real time. From the practical point of view the influence of road profile on vehicle vibration and tire forces is interested. PSD of road unevenness has one dominant peak at frequency f = 0,122 Hz. This frequency correspond the wave length L = 81,97 m. It signalizes the very good road profile. This frequency has dominant position also in PSD of vehicle response. PSD of tire force has two zones of dominant frequencies, between 1 – 2 Hz and 8 – 12 Hz. These frequency zones correspond to natural frequencies of vehicle (1,13; 1,29; 1,45; 8,89; 10,91; 11,71 Hz). PRF of tire force has one dominant peak corresponding the frequency 8,89 Hz. This is the natural frequency coupled with wheel natural vibration.

Symbols

[m], [b], [k] – mass, damping and stiffness matrix; $\{r\}, \{\dot{r}\}, \{\ddot{r}\}$ – vectors of displacement, speed and acceleration; $\{F\}, \{F_d\}, \{F_{re}\}, \{F_v\}$ – force vectors; Ω – wave number [rad/m]; h – road unevenness; x – length coordinate; φ_i – phase angle; $x(k)$ – real number; $X(k)$ – complex number; i, I, n, N, k – natural numbers; e – Euler number; i – complex unit, f – frequency, *PA* – Peak Amplitude; *RMS* – Root Mean Square Amplitude; *AS* – Auto Spectrum; *PS* – Power Spectrum; *PSD* – Power Spectral density; *FS* – Phase Spectrum; *W* – weighting function; $H(\mathrm{i}f)$ – Frequency Response Function; *PRF* – Power Response Factor; L – wave length.

Literature

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OHLAS VOZIDLA VO FREKVENČNEJ OBLASTI

Resumé

Predkladaný príspevok je venovaný metódam numerického modelovania pohybu vozidiel po cestných komunikáciách. Uvažuje priestorový výpočtový model vozidla Tatra 815 a výpočtový model tuhej vozovky s náhodným profilom povrchu jazdnej dráhy. Prezentované sú výsledky numerického riešenia pohybu vozidla po vozovke vo frekvenčnej oblasti v tvare výkonových spektrálnych hustôt. Numerické výpočty sú robené v prostredí programovacieho jazyka vyššej úrovne MATLAB. Získané výsledky sú zobrazené v grafickej forme.