ROCZNIKI INŻYNIERII BUDOWLANEJ – ZESZYT 15/2015
Komisja Inżynierii Budowlanej Oddział Polskiej Akademii Nauk w Katowicach

COMPARISON OF ANALYTICAL AND THE FEM METHOD FOR DERIVING THE MODE I STRESS INTENSITY FACTOR OF THIN-WALLED BEAMS CONTAINING A CRACK

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Keywords: *Cracked thin-walled beams, Mode I SIF, Cross-section warping, FEM, J-EDI.*

1. Introduction

At present, there is a trend in constructions design and applications to use the lightweight structures. This is why the thin-walled beams are being widely applied in modern engineering structures. Analyzing the crack behavior in such structural elements is extremely important. The Mode I stress intensity factor (SIF) is an important parameter for the structural integrity since it characterizes the stress concentration. The exact determination of the SIF generally is a difficult task. In the case of the thin-walled beams, appearance of the cross section warping is further aggravating its calculations.

The problem of determining the stress intensity factor of cracked structural elements was the subject of numerous researches in the past. Kienzler and Herrmann [1] were determining the SIF for the cracked beam of a rectangular cross-section, by application of the energy conservation law and the elementary beam theory, for the case of pure bending. Herrmann and Sosa [2] used such a method to determine the SIF of the cracked rectangular bar loaded in tension, with the stiffness discontinuity and of a bar with two different rectangular cross-sections. Gao and Herrmann [3] have shown that the SIF can be obtained through asymptotic matching with standard limiting crack solutions. Using the same method Müller, et al. [4] have determined the SIFs of cracked beams of circular and rectangular cross-sections for pure loading in bending, tension and torsion. Ricci and Viola [5] used the Kienzler and Herrmann [1] method for determination of the SIFs of the cracked beam with the T cross-section and they considered the dynamic behavior of such beams, as well.

An analytical method for determination of the stress intensity factor of the thin-walled beams with a crack, subjected to the bending moment and the axial force (Fig. 1(a)) is presented in this paper. The classical expressions for the SIFs, proposed by Tada et al. [6]

for thin plates, are applied. The influence of the cross-section warping, a phenomenon linked to thin-walled structures, was taken into account, as well. The proposed analytical method was verified by comparison to values for the stress intensity factors obtained by numerical simulation, done by the Finite Element Method (FEM), by use of the J-EDI method and the software package PAK-FK&F, Živković et al. [7].

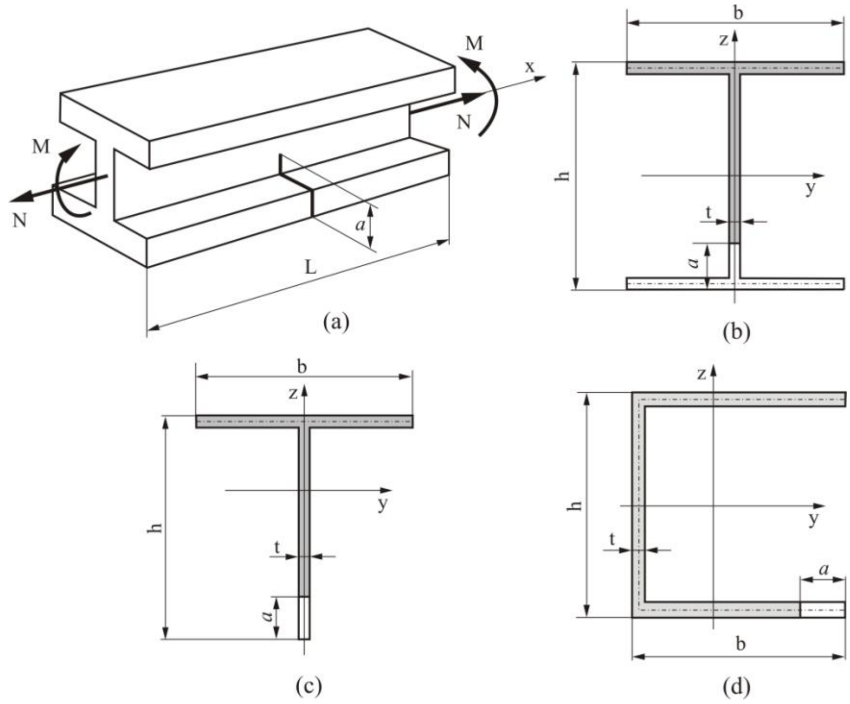

Fig. 1. (a) Geometry and loading of the cracked thin-walled beam;
(b) I-profile; (c) T-profile and (d) U-profile.

2. Determination of the Stress Intensity Factor

2.1. Analytical model

The considered beam, subjected to the bending moment and the axial force, and the corresponding cross-sections, are shown in Fig. 1. The cross-sections' dimensions, described in Fig. 1, are the flange width *b* and the web height *h*. The thickness of the flange and the web is the same, *t* and the crack length is *a*. To determine the stress intensity factor, the flange (or the web) with a crack are considered separately, independently of the rest of the beam.

Expression for the Mode I stress intensity factor for the thin plate, K_I, for the case of loading with the axial force and the bending moment is the sum of classical expressions for stress intensity factors for the two loadings, proposed by Tada et al. [6]:

$$K_I = K_I^{(N)} + K_I^{(M)}, \tag{1}$$

where:

$$K_I^{(N)} = \frac{N_T}{tb}\sqrt{\pi a}\left[0.265(1-a/b)^4 + \frac{0.857+0.265a/b}{\sqrt{(1-a/b)^3}}\right] , \quad (2)$$

$$K_I^{(M)} = \frac{6M_T}{tb^2}\sqrt{\frac{2b}{\pi a}\tan\frac{\pi a}{2b}}\left[\frac{0.923+0.199(1-\sin(\pi a/2b))^4}{\cos(\pi a/2b)}\right] , \quad (3)$$

are the stress intensity factors for the normal axial force and the bending moment, respectively, and where N_T and M_T are the reduced axial force and the bending moment, respectively, which act upon the plate of width b and thickness t. For the case of the thin-walled beam N_T and M_T are given, respectively, by Cortinez and Dotti [8]:

$$N_T = t\left(\frac{N}{A}\int ds + \frac{M_y}{I_y}\int zds + \frac{M_z}{I_z}\int yds + \frac{B}{I_\Omega}\int \Omega ds\right), \quad (4)$$

$$M_T = t\left(\frac{N}{A}\int sds + \frac{M_y}{I_y}\int szds + \frac{M_z}{I_z}\int syds + \frac{B}{I_\Omega}\int s\Omega ds\right), \quad (5)$$

where: N – is the axial force, M_y and M_z – are the bending moments about the y and the z axis, respectively, B – is the bimoment, A – is the cross-section area, I_y and I_z are the second area moments (moments of inertia) for axes y and z, respectively. Here ω_P – is the sectorial coordinate, I_Ω – is the sectorial moment of inertia and s – is the arc coordinate. Warping is taken into account by considering the contribution by the bimoment.

2.2. Numerical model

The main objective for the Finite Element Method (FEM) application in the fracture mechanics is determination of the stress field around the crack tip. The most important problem is to determine the level of the stress concentration, which is expressed by the stress intensity factor. The J-EDI (Equivalent Domain Integral) method is based on theory of the virtual crack extension. The discretized form of the J-EDI integral, in the 2D space can be written in the form, Jovicic et al. [9]:

$$J_k = \sum_{\substack{elements\\ in\ A}}\sum_{p=1}^{P}\left[\left(\sigma_{ij}\frac{\partial u_i}{\partial X_k} - W\delta_{kj}\right)\frac{\partial q}{\partial X_j}\det\left(\frac{\partial X_m}{\partial \eta_n}\right)\right]_p w_p\ , \quad i, j, k, m, n = 1,2, \quad (6)$$

where: P – is the number of interpolation points per element, w_p – is the weighting factor, σ_{ij} – are components of the stress tensor, u_i – are the displacement vector's components, W – is the specific strain energy, δ_{kj} – is the Kronecker's delta symbol, X_i – are the global displacement components and η_i – are the local coordinates.

For the 3D problems, the discretized form of the J-EDI integral can be written in the form, Jovicic et al. [9]:

$$J_k = \frac{1}{f} \sum_{\substack{elements \\ in\,V}} \sum_{p=1}^{P} \left[\left(\sigma_{ij} \frac{\partial u_i}{\partial X_k} - W \delta_{kj} \right) \frac{\partial q}{\partial X_j} \det\left(\frac{\partial X_m}{\partial \eta_n} \right) \right]_p w_p \,, \quad i, j, k, m, n = 1,3, \qquad (7)$$

where f = (2/3)Δ and Δ denotes the 3D element thickness in the crack front direction. Relations (6) and (7) are given for the structures made of homogeneous materials, without presence of body forces.

The procedure for automatic searching over the discretized space of finite elements, as well as for the definition of the rigid region, and assigning the corresponding values of the weight function to nodes that belong to that region, was developed within the software package PAK-FK&F, Živković et al. [7].

The thin-walled profiles presented in Figs. 1(b) and 1(c) are symmetrical, thus only one half of the profile was modeled. The following dimensions of the profiles were taken for analysis: h=0.2 [m], b=0.1 [m], t=0.01 [m] and L=2 [m]. The loads were the following: the normal force N=6 [kN], the bending moment M=6 [kNm] and the bimoment B=341 [Nm^2]. Material used in analysis is steel St 52-3, with the following characteristics: Young's elasticity modulus E=210 [GPa], Poisson's ratio ν=0.3 and the fracture toughness K_{IC}=158 [MPa·m$^{1/2}$].

3. Results and discussion

In Figs. 2, 3 and 4 variations of the stress fields at the crack tip of the thin-walled beams with the crack length are shown, determined by the PAK-FM&F software package, for the I, T and U profiles, respectively (stress values are in given in MPa).

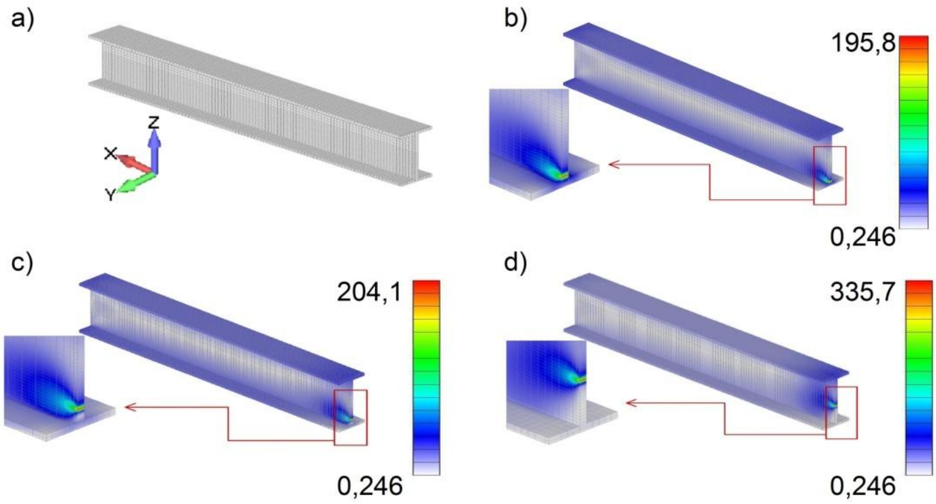

Fig. 2. The FEM model (a) and the stress field of the I-profiled thin walled beam for different crack lengths (b) a=15 [mm], (c) a=30 [mm], and (d) a=80 [mm]. Stresses are given in MPa.

In Figs. 5-7 diagrams of the Mode I stress intensity factor variation with the normalized crack length obtained by the analytical method (solid line) and by the numerical simulation by the FEM software package PAK-FM&F (broken line) are presented. In all three figures, the same trend can be noticed. The slope of the curves, obtained by both methods, changes when the crack passes from the flange to the web. The curves' slope is steeper on the flange than on the web.

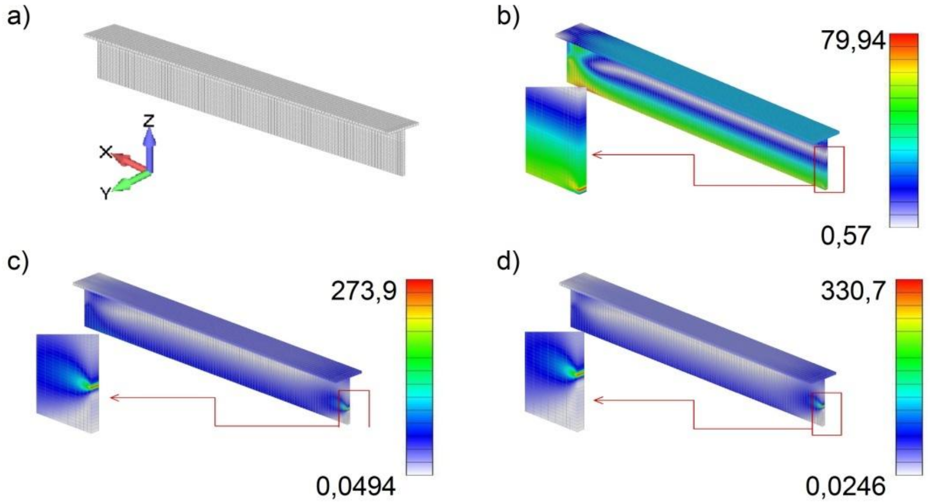

Fig. 3. The FEM model (a) and the stress field of the T-profiled thin walled beam for different crack lengths (b) *a*=5 [mm], (c) *a*=65 [mm] and (d) *a*=80 [mm]. Stress are given in MPa.

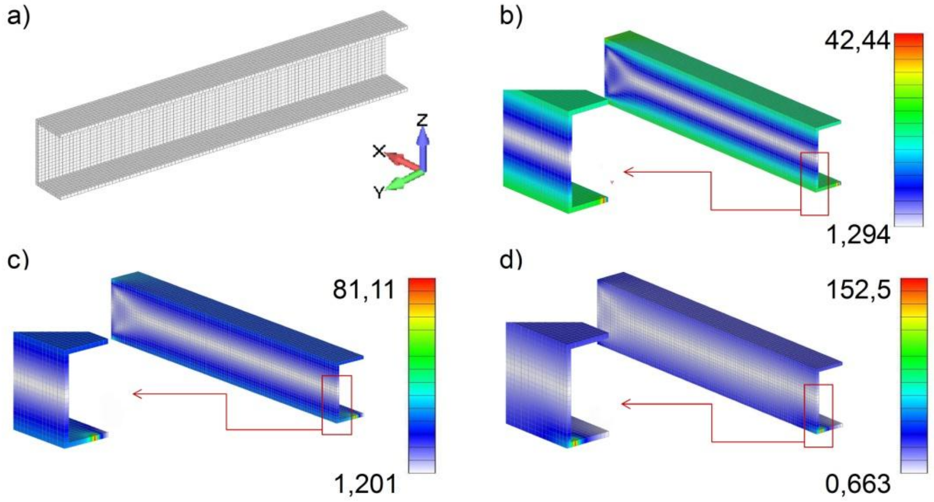

Fig. 4. The FEM model (a) and the stress field of the U-profiled thin walled beam for different crack lengths (b) *a*=10 [mm], (c) *a*=60 [mm] and (d) *a*=80 [mm]. Stresses are given in MPa.

From Figs. 5-7 one can notice that the analytical and numerical solutions agree well for the small crack lengths, approximately up to a/h=0.2. For the longer cracks, the difference between the two curves becomes significant. This means that the analytical expressions for the Mode I stress intensity factors, can be used for the small crack lengths, only.

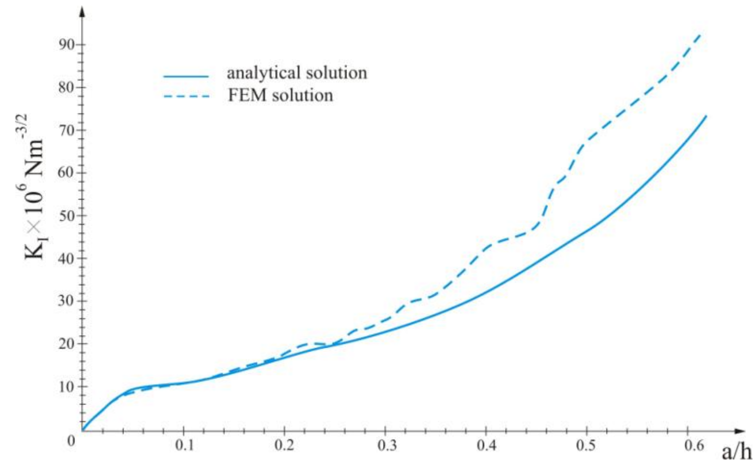

Fig. 5. Mode I stress intensity factor in terms of the normalized crack length for the beam with the I cross-section.

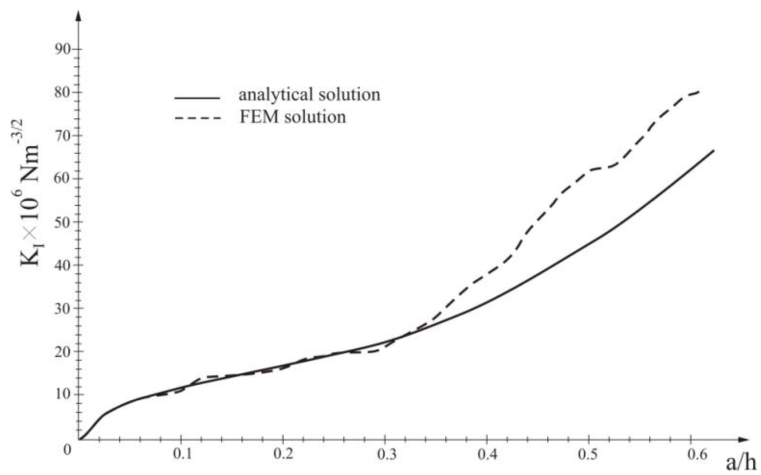

Fig. 6. Mode I stress intensity factor in terms of the normalized crack length for the beam with the T cross-section.

4. Conclusions

An analytical method for determination of the Mode I stress intensity factor for the cracked thin walled beams, of the I (H), T and U cross-sections, is presented in the paper. The beams are subjected to axial force and the bending moment. This analytical method is based on application of the classical expression for the stress intensity factor for thin plates, extended to thin profiles, with taking into account the cross-section warping. The numerical simulation was done by the Finite Element Method with the software package PAK-FM&F.

The good agreement of results obtained by the analytical and numerical method was noticed for the small crack lengths. However, for the longer cracks, the difference between the analytical and the FEM simulation results becomes significant.

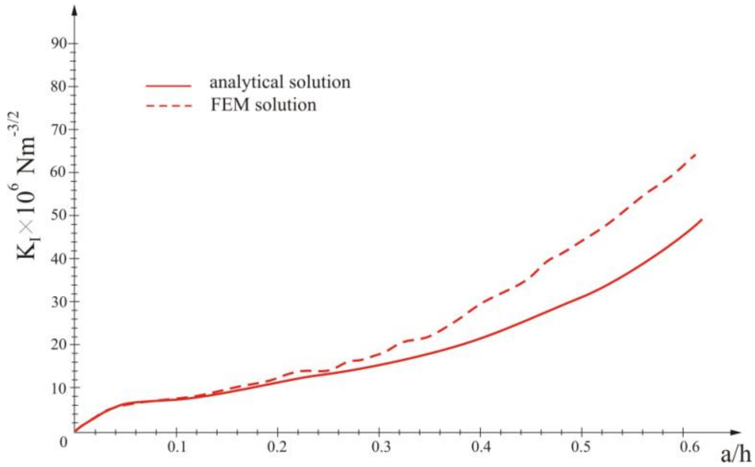

Fig. 7. Mode I stress intensity factor in terms of the normalized crack length for the beam with the U cross-section.

That leads to the conclusion that the analytical expressions can be used for the small crack length, only. However, the cracks that are longer than 20 % of the profiles' heights would be considered as critical, probably causing the fracture of the whole beam. Thus, for the practical purposes of structural design, the proposed analytical method provides good estimates.

Acknowledgements

This research was partially supported by the Ministry of Education, Science and Technological Development of Republic of Serbia through Grants ON174001, ON174004 and TR32036 and by the European regional development fund and Slovak state budget by the project "Research Center of the University of Žilina" - ITMS 26220220183**.**

Note: *Results presented in this paper were partially presented at the Conference "Static of buildings", 12.-13. March 2015., Pieštany, Slovakia – reference [10].*

Denotations of symbols

A – cross-sectional area, [m^2],
B – bimoment, [Nm^2],
E – Young's elasticity modulus, [Pa],
I_y, I_z – cross sectional moment of inertia for the y and z axes, respectively, [m^4],

I_Ω – sectorial cross-sectional moment of inertia, [m^6],
K_I – stress intensity factor of the thin plate for the Mode I crack propagation, [Nm$^{-3/2}$],
$K_I^{(N)}$ – stress intensity factor of the thin plate due to normal force for the Mode I loading, [Nm$^{-3/2}$],
$K_I^{(M)}$ – stress intensity factor of the thin plate due to bending moment for the Mode I loading, [Nm$^{-3/2}$],
M – bending moment, [Nm],
M_T – reduced bending moment, [Nm],
M_y, M_z – bending moments about y and z axes, respectively, [Nm],
N – normal axial force, [N],
N_T – reduced normal force, [N],
P – number of interpolation points per element, [-],
V – volume, [m^3],
W – specific strain energy, [N/m^2],
X_i – global displacement components, [m],
a – crack length, [m],
b – flange width, [m],
h – web height, [m],
s – arc coordinate, [m],
t – flange and web thickness, [m],
u_i – displacement vector components, [m],
y, z – coordinates, [m],
w_p – weighting factor, [-],
Δ – thickness of the 3D element, [m],
Ω – normalized sectorial coordinate, [m^2],
δ_{ij} – Kronecker's delta symbol,
η – local coordinates, [m],
σ_{ij} – stress tensor components, [Pa],
ω_p – sectorial coordinate, [m^2].

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Summary

The problem considered in this paper is of the cracked thin-walled beams. They are subjected to combined loading by the bending moment and the axial force. The profiles with the I (H), T and U cross-sections were analyzed. The analytical method is based on application of classical expressions for Mode I SIF of such beams with taking into account the cross sections' warping. The classical expressions for SIF for the thin plate are adapted by including the reduced forces and moments characteristic for the thin-walled beams. The numerical simulation for calculating the stress intensity factor of the cracked thin-walled beams consisted of application of the Finite Element Method (FEM) by use of the J-EDI method. It is found that the Mode I SIF of cracked thin-walled beams increases with crack length increase. The conducted comparison of results obtained by application of the two methods confirms the validity of the proposed analytical expressions only for the small crack lengths.