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# HOMOGENIZATION TECHNIQUES OF UNIDIRECTIONAL COMPOSITE MATERIAL

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## 1. Introduction

A usage of various homogenization techniques is an alternative to the experimental determination of material properties of fiber matrix composite material. Many analytical homogenization techniques are based on the equivalent eingenstrain method, which considers the problem of a single inclusion embedded in an infinite elastic medium. Homogenization has been accomplished by using various techniques including the Fourier series technique, variational principles etc. Most fiber matrix composites have random arrangement of the fibers [1] (Fig. 1).

Fig. 1. Randomly distributed fibers.

## 2. Mori-Tanaka method

In the last decade, effective media theories, widely used in classical continuum micromechanics, have been recognized as an attractive alternative to Finite Element Analysis (FEA) based methods. Since its introduction the Mori-Tanaka (MT) method has enjoyed a considerable interest in a variety of engineering applications. These include classical fiber matrix composites too [2].

General description of the Mori-Tanaka method in the framework of elasticity is treated in this section. The Mori-Tanaka method takes into account the effect of phase interactions on the local stresses by assuming an approximation in which the stress in each phase is equal to that of a single inclusion *r* embedded into an unbounded matrix subjected to as yet unknown average strain or stress matrix [2].

The constitutive equation **σ**=**C**·**ε** can be written in the following matrix form:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{Bmatrix}=\begin{bmatrix} n & l & l & 0 & 0 & 0\\ l & (k+m) & (k-m) & 0 & 0 & 0\\ l & (k-m) & (k+m) & 0 & 0 & 0\\ 0 & 0 & 0 & m & 0 & 0\\ 0 & 0 & 0 & 0 & p & 0\\ 0 & 0 & 0 & 0 & 0 & p\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{Bmatrix}. \tag{1}$$

Material characteristics can be determined from the following equations:

$$k=-\left(1/G_{23}-4/E_{22}+4\nu_{12}^2/E_{11}\right)^{-1},\quad l=2k\nu_{12},\quad m=G_{23},$$
$$n=E_{11}+4k\nu_{12}^2=E_{11}+l^2/k,\qquad p=G_{12}. \tag{2}$$

## 3. Periodic analytical model

If the composite has periodic microstructure, then Fourier series can be used to estimate all the components of the stiffness tensor of the composite. Explicit formulas for a composite reinforced by long circular cylindrical fibers, which are periodically arranged in a square array, are written in the following way [3]:

$$C_{11}=\lambda^{(m)}+2\mu^{(m)}-\frac{\xi}{D}\left(\begin{array}{l}\dfrac{S_3^2}{\mu^{(m)2}}-\dfrac{2S_6S_3}{\mu^{(m)2}g}-\dfrac{aS_3}{\mu^{(m)}c}+\dfrac{S_6^2-S_7^2}{\mu^{(m)2}g^2}+\\ +\dfrac{aS_6+bS_7}{\mu^{(m)}gc}+\dfrac{a^2-b^2}{4c^2}\end{array}\right),$$

$$C_{12}=\lambda^{(m)}+\frac{\xi}{D}b\left(\frac{S_3}{2c\mu^{(m)}}-\frac{S_6-S_7}{2c\mu^{(m)}g}-\frac{a+b}{4c^2}\right),$$

$$C_{22}=\lambda^{(m)}+2\mu^{(m)}-\frac{\xi}{D}\left(-\frac{aS_3}{2\mu^{(m)}c}+\frac{aS_6}{2\mu^{(m)}gc}+\frac{a^2-b^2}{4c^2}\right),$$

$$C_{66}=\mu^{(m)}-\xi\left(-\frac{S_3}{\mu^{(m)}}+\left(\mu^{(m)}-\mu^{(f)}\right)^{-1}\right)^{-1},\quad C_{23}=\lambda^{(m)}+\frac{\xi}{D}\left(\frac{aS_7}{2\mu^{(m)}gc}-\frac{ba+b^2}{4c^2}\right),$$

$$C_{44}=\mu^{(m)}-\xi\left(\frac{2S_3}{\mu^{(m)}}+\left(\mu^{(m)}-\mu^{(f)}\right)^{-1}+\frac{4S_7}{\mu^{(m)}\left(2-2\nu^{(m)}\right)}\right)^{-1}, \tag{3}$$

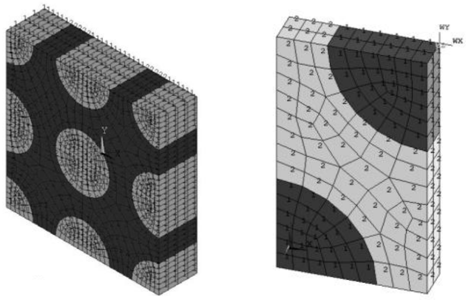
where: *D*, *a*, *b*, *c*, *g* and $S_i$ (*i*=3,6,7) for composite reinforced by long circular cylindrical fibers can be found in [3].

Assuming the fiber and matrix are both isotropic, Lame constants of both materials are obtained by:

$$\lambda = \frac{E}{(1+\nu)(1-2\nu)}, \quad \mu = G\,. \tag{4}$$

## 4. Numerical homogenization

A random microstructure results in transversely isotropic properties at the meso-scale. The analysis of composites with random microstructure can be done by using of a fictitious periodic microstructure (Fig. 2a). A simple alternative is to assume that the random microstructure is well approximated by the hexagonal microstructure (Fig. 2b).



a) b)

Fig. 2. a) Hexagonal microstructure FEA quarter model, b) Periodic square microstructure FEA model.

In order to evaluate the elastic matrix **C** of the composite, the Representative Volume Element (RVE) is subjected to an average strain. The volume average of the strain in the RVE equals the applied strain [4,5]:

$$\bar{\varepsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij} dV \; . \tag{5}$$

The components of the tensor **C** are determined by solving elastic models of RVE with parameters ($a_1$, $a_2$, $a_3$) subjected to the boundary conditions. By using a unit value of applied strain, it is possible to compute the stress field, whose average gives the required components of the elastic matrix as [4]:

$$C_{ij} = \bar{\sigma}_i = \frac{1}{V}\int_V \sigma_i dV \quad \text{for} \quad \varepsilon_j^0 = 1\,. \tag{6}$$

The coefficients in **C** are found by setting a different problem for each column of **C**.

## 5. Example of homogenization

In the example, the material characteristics $E_1$, $E_2$, $\nu_{12}$, $\nu_{23}$ and $G_{12}$ were computed for a unidirectional composite with isotropic fibers ($E_f$=230 [GPa], $\nu_f$=0.3) and isotropic matrix ($E_m$=3.2 [GPa], $\nu_m$=0.4). The fiber volume fraction and the fiber diameter were assumed as $\xi$=0.6 and $d_f$=7 [μm].

## 6. Conclusions

The paper compares analytical and numerical approaches of homogenization of unidirectional lamina consists of fiber matrix composite material. In the frame of numerical homogenization the best results were obtained from the hexagonal microstructure model. In the frame of analytical homogenization the best results were obtained from the periodic microstructure model. These two models are suitable for analytical approach of modelling of unidirectional lamina [6]. The results obtained from these models are very similar.

The example is solved by programs Heat and Elasticity Properties (Mori-Tanaka method) [1], program MATLAB (Periodic analytical model) and program ANSYS with the help of Finite Element Method. The elastic properties of the homogenized material are shown in the Table 1.

Table 1. Summary of results.

| | Periodic numerical model | Hexagon. numerical model | Periodic analytical model | Mori Tanaka model |
|---|---|---|---|---|
| $E_1$ [GPa] | 139.378 | 139.355 | 139.291 | 139.295 |
| $E_2$ [GPa] | 20.454 | 14.008 | 14.262 | 10.998 |
| $\nu_{12}$ [-] | 0.33 | 0.36 | 0.36 | 0.26 |
| $\nu_{23}$ [-] | 0.33 | 0.52 | 0.51 | 0.49 |
| $G_{12}$ [GPa] | 4.771 | 4.391 | 4.407 | 4.265 |

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## Denotations of symbols

**C** – elasticity tensor, [Pa],
$D$, $a$, $b$, $c$, $g$, $S_i$ – coefficients for composite reinforced by long circular cylindrical fibers,
$E$ – modulus of elasticity, [Pa]
$E_{11}$ – modulus of elasticity in longitudinal direction, [Pa],
$E_{22}$ – modulus of elasticity in transversial direction, [Pa],
$G$ – shear modulus of elasticity, [Pa],
$G_{12}$ – in-plane shear modulus, [Pa],

$G_{23}$ – transverse shear modulus, [Pa],
$\boldsymbol{\varepsilon}$ – strain tensor, [-],
$\lambda^{(m)}$, $\mu^{(m)}$ – Lamé constants of matrix, [Pa],
$\lambda^{(f)}$, $\mu^{(f)}$ – Lamé constants of fibers, [Pa],
$\nu$ – Poisson ratio, [-],
$\nu_{12}$ – in-plane Poisson ratio, [-],
$\nu_{23}$ – transverse Poisson ratio, [-],
$\boldsymbol{\sigma}$ – stress tensor, [Pa],
$\xi$ – fiber volume fraction, [-].

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### Summary

The paper deals with analytical and numerical homogenization of unidirectional fiber matrix composite. There are described the Mori-Tanaka method, periodic analytical model and numerical periodic and hexagonal models. The example of homogenization is solved by programs Heat and Elasticity Properties, MATLAB and ANSYS. The obtained results are summarized in the table and compared to each other.